Go to Volume 2

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Uncertainty Study of INEEL EST Laboratory Battery Testing Systems

Volume 1

Background and Derivation of Uncertainty Relationships

John L. Morrison
Gary L. Hunt
Donna J. Marts
Chester G. Motloch

Idaho National Engineering and Environmental Laboratory
Bechtel BWXT Idaho, LLC

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Idaho National Engineering and Environmental Laboratory
Idaho Falls, Idaho 83415

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ABSTRACT

The Idaho National Engineering and Environmental Laboratory has been performing tests of high-power batteries for application in electric and hybrid vehicles for various development programs since 1983. The parameters important to these tests are either directly measured or derived from the direct measurements. The program managers of the sponsoring programs expressed a need to understand the confidence that could be placed in the results of this testing, thus the uncertainty of the parameters was investigated. This report summarizes the INEEL high-power battery testing process and presents the complete derivation of uncertainty for every parameter of interest.

This uncertainty study addresses the derivation of the analytical expressions for both the measured and derived parameters as well as the calculated uncertainty values. Since the calculated values are hardware specific, the study has been separated into multiple volumes to facilitate presentation of the information. Volume 1 summarizes the INEEL high-power battery testing process and presents the complete derivation of uncertainty for every measured and derived parameter of interest. A separate volume will be issued addressing the various test stations used at the INEEL and will present the actual uncertainty values associated with each.

CONTENTS

FIGURES

TABLES

ACRONYMS

% FS	percentage of full scale
% Rdg	percentage of reading
ΔI	current change (between two defined time points)
ΔV	voltage change (between two defined time points)
A·h	ampere-hour
ATD	Advanced Technology Development (Program)
dB	decibel
dc	direct current
DOE	U.S. Department of Energy
DVM	digital voltmeter
EST	Energy Storage Testing (Laboratory)
HPPC	Hybrid Pulse Power Characterization (Test)
I_{avg}	average current (over some defined interval)
INEEL	Idaho National Engineering and Environmental Laboratory
LSB	least significant bit
PNGV	Partnership for a New Generation of Vehicles
ppm	parts per million
SD	self-discharge
sps	samples per second
STD	standard deviation
TC	thermocouple
V_{avg}	average voltage (over some defined interval)
W·h/d	watt-hours per day

Uncertainty Study of INEEL EST Laboratory Battery Testing Systems

Volume 1 Background and Derivation of Uncertainty Relationships

1. INTRODUCTION

1.1 Background of the Energy Storage Testing Laboratory Battery Testing Program

The Energy Storage Testing (EST) Laboratory at the Idaho National Engineering and Environmental Laboratory (INEEL) was established in 1983 for testing full-size electric vehicle batteries in support of the U.S. Department of Energy's (DOE's) Electric and Hybrid Vehicle Program. The laboratory's original mission was later expanded to include other types of energy storage and conversion devices, though battery testing remains its principal focus. Presently, the laboratory serves as an independent test facility for testing hybrid vehicle battery technologies in various development programs. These include the DOE-sponsored Advanced Technology Development (ATD) Program, which seeks to improve the electrochemical performance of lithium ion cells, and the Partnership for a New Generation of Vehicles (PNGV) Program (now being replaced by the FreedomCAR Program), which is jointly sponsored by DOE and U.S. automobile manufacturers. The spectrum of devices tested for these programs ranges from small research cells to full-size battery systems, using one of several electrochemical technologies such as lithium ion, lithium polymer, or nickel metal hydride. The common element for this testing is that all these batteries are designed to provide very high power levels relative to their weight and volume, with only modest energy requirements compared to conventional electric vehicle requirements.

1.2 Structure of This Report

Because the INEEL EST Laboratory performs testing on a continuous basis, access to battery test stations for this study was limited to times when the various testers could be used without programmatic impact on work in progress. As a result, the testing parts of the study are necessarily performed over an extended period. Consequently, the report documenting the measurement uncertainty study is being published in multiple volumes, as the test data and subsequent analysis results become available.

This first volume of the overall report describes the INEEL high-power battery testing process and the general accuracy specifications of the various types of battery test stations considered in the study. The primary technical content of this volume is a derivation of the relationships for determining measurement uncertainty using some combination of manufacturer's specifications and confirmatory testing. These are derived first for generic parameters and then for the specific parameters of interest to INEEL battery testing. Specific parameters include both directly measured variables (temperature, current, and voltage) and the various parameters that are indirectly derived from them for the PNGV and ATD testing programs. These relationships provide the key for all subsequent measurement uncertainty analysis, and they can be applied to any similar measurement process as required. A derivation of the potential measurement uncertainty effects of signal aliasing is also included in this volume.

No equipment-specific numerical results are included. The application of these relationships to specific types of INEEL battery test stations will be documented in one or more subsequent volumes of this report, which will describe their measurement uncertainty characteristics based on their design, manufacturer's specifications, and various experiments conducted to verify or supplement manufacturer's information.

Note that all relationships developed in this analysis (and subsequently applied in the following volumes) are expressed in terms of a standard deviation and as such have a confidence level of about 67%. If a specific application requires a higher confidence level, then the appropriate adjustment factor must be applied. For example, a factor of two (i.e., two standard deviations) is commonly used to give a 95% confidence level, though even higher values are sometimes necessary for truly critical applications.

1.3 Overview of the High-Power Battery Testing Process

The ATD and PNGV programs have defined slightly different approaches to battery testing. This is largely due to the fact that ATD is a research-oriented program, and PNGV aims to make use of available commercially viable battery designs in a multiphase development process. As a result of these orientations, the ATD program is conducted largely within the DOE national laboratories. The PNGV program contracts all its development effort to commercial battery manufacturers, with INEEL providing independent testing to verify the contractor's progress and results. Many of the elements of the battery testing process are the same for both programs. The largest difference is that the ATD program is interested primarily in relative improvements in performance, while PNGV is concerned with whether the technologies it sponsors will be able to meet its performance goals.

In either case, the primary focus of high-power battery testing for hybrid vehicle applications has traditionally been on power performance, though there are a number of other factors that are also very important. The nature of hybrid vehicle design is such that many of these factors interact, i.e., improving one of them often leads to compromising one or more of the others. As a result, the PNGV program is designed around a set of interrelated goals derived from vehicle requirements and constraints. All of these goals are intended to be satisfied simultaneously by a successful battery. The associated PNGV testing process uses custom test procedures to measure various aspects of battery performance for direct comparison with these goals.[a] These procedures typically subject a device under test to a prescribed sequence of controlled current, power, or voltage steps, each lasting from a few seconds to an hour or more. The ATD program has also adopted many of these procedures (or variants of them) as useful means for comparing high-power cell performance.

Many of the battery performance results important to these programs are not directly measured as simple parameters. Instead, they must be calculated from the results of various time-based measurements made under particular test conditions. The program managers of the sponsoring programs expressed a need to understand the confidence that could be placed in the results of this testing, specifically with respect to the uncertainty of the reported results. This concern led directly to the measurement uncertainty evaluation detailed in this report.

a. See the *PNGV Battery Testing Manual*, Revision 3, DOE/ID-10597 (published February 2001) for a description of these test procedures and the PNGV program goals on which they are based.

1.4 Parameters Important to ATD and PNGV Testing

This section defines the parameters important to INEEL hybrid vehicle battery testing, so that the measurement uncertainty expressions to be developed can be understood in context. They are divided into two groups: directly measured parameters (which are typically not of interest for themselves) and the various parameters derived from these fundamental measurements. The PNGV goals are defined for full-size battery systems, but most PNGV and ATD testing is done on single-cell devices. The results of cell testing are extrapolated to full-size batteries, and some system-level parameters of concern (such as thermal control system losses) are not treated in this report.

1.4.1 Measured Parameters

For a single-cell device, only three fundamental measurements are typically made as functions of time over a prescribed test sequence: the *temperature* of the cell and the terminal *voltage* and *current* during the various load conditions imposed on the cell.

Cell temperature itself is not a goal parameter (except for the ability to operate over a defined temperature range), but cell performance of high-power batteries can vary dramatically with temperature. Most cell testing is done in test chambers at ambient temperature (typically 25 to 30°C), and accurate control of temperature is critical to repeatable results. Additionally, the characterization of temperature effects on cell performance requires accurate measurement of the device temperatures during tests conducted at higher- or lower-than-normal temperatures.

All three of these parameters are typically measured at periodic intervals during a battery test sequence, though the measurement rate may be different during different parts of the sequence.

1.4.2 Derived Parameters for High-Power Battery Testing

Batteries are robust energy storage devices whose use may involve time responses from near steady state to fractions of seconds. Various parameters of battery performance are derived for the ATD and PNGV programs from the fundamental measurements of current and voltage made during specific test sequences, i.e., under transient conditions. These parameters have somewhat generic names, but they (and their units) are defined in the context of these programs and this report, as follows.[b]

1.4.2.1 Power. *Battery Power* is the instantaneous product of current and voltage and is typically expressed in watts. Battery voltage is always positive, but current can flow either out of the battery (during discharge) or into the battery (during charge), so power can be either positive or negative. Battery testers typically perform this calculation internally and report the result as measured data, though it is usually not "measured" in the sense that it is derived from the output of an actual power sensor. In some cases, power must be calculated externally from the measured values of current and voltage. The ability to do this usefully for step (pulse) test profiles can critically depend on the extent to which the current and voltage measurements are made at the same time. Careful design of battery test time profiles is required to ensure that the time resolution of transient measurements is always adequate.

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b. Expressions used for calculating these parameters are shown in the equations presented in Section 3.2.

1.4.2.2 Capacity. *Battery Capacity* is a measure of [net] charge removed from a battery, defined as the integral of battery current over time during some prescribed test sequence.[c,d] Capacity is typically expressed in ampere-hours (A·h).[e] Battery testers commonly perform this integration internally (typically as a numerical integration) and report the result as measured data, though it can also be calculated externally if a sufficient number of data points are available.

1.4.2.3 Energy. *Battery Energy* is a measure of the energy removed from or added to a battery, defined as the integral of battery power over time under some prescribed test sequence. Energy is typically expressed in watt-hours (W·h). Battery testers commonly perform this integration internally (typically as a numerical integration) and report the result as measured data, though it can also be calculated externally if a sufficient number of data points are available.

1.4.2.4 Source Impedance. *Source Impedance*, expressed in ohms (Ω), is a measure of the apparent relationship between battery terminal voltage (V) and battery load current (I) over a selected portion of a particular test step, which is commonly a constant current pulse several seconds in duration. It is calculated for a test pulse as *Source Impedance* = $\Delta V/\Delta I$, where ΔV and ΔI are respectively the change in voltage resulting from the change in current over some or all of the test pulse. The test pulse is commonly preceded by an open-circuit condition. The sign of battery current is defined such that this quantity is always positive.

1.4.2.5 Efficiency. *Battery Round-Trip Efficiency* is the ratio of discharge energy to charge energy (expressed as a percentage) over a specific test sequence where the initial and final battery states-of-charge are identical. This test sequence is typically one or more pulse “profiles” (sequences of discharge and charge steps) controlled such that the discharge capacity removed is equal to the charge capacity returned during each profile.

1.4.2.6 Self-Discharge. *Battery self-discharge* is the amount of battery energy lost [typically expressed in watt-hours per day (W·h/d)] during a fixed time period when the battery is stored in an open-circuit condition. It is calculated from the results of two nearly identical tests, each of which discharges the battery at a constant current rate from a fully charged state to a minimum terminal voltage. The tests differ only in that the second test is interrupted at an intermediate state, where the battery is allowed to stand in an open-circuit state for a period of (nominally) seven days before the discharge is resumed. The difference between the battery discharge energy values measured during the two tests is considered *self-discharge*.

1.4.2.7 Pulse Power Capability. *Pulse power capability* is a calculation of the maximum power [typically expressed in watts (W)] that can be delivered or accepted by a battery at a given depth-of-discharge for a prescribed time without exceeding prescribed voltage limits. It is calculated from voltage measurements taken before and during (and sometimes after) execution of a current pulse, along with current measurements taken before and during the pulse.

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c. Note that the term *charge* is used in this sense only in this definition. Elsewhere in this report, *charge* refers to the act of adding capacity to a battery, while *discharge* similarly refers to removing capacity.

d. Common test sequences include constant-current or constant-power discharge to a prescribed minimum terminal voltage, though both capacity and energy are commonly measured during part or all of various step (pulse) test sequences as well.

e. One ampere-hour is 3600 ampere-seconds (A·s), or 3600 coulombs (C).

1.5 General Functional Description of a Battery Test and Measurement System

The high-power battery test systems used at the INEEL are made by a number of different manufacturers and include a wide range of voltage, current, and power capabilities. Some are designed to test a single battery, while others are capable of testing 16 or more cells simultaneously. However, most share a number of common characteristics relevant to measurement uncertainty considerations. This frequent changes between these states. Depending on the test to be performed, it may be necessary to maintain a constant current, voltage, or power level at the battery terminals in either the discharge or charge state. To accomplish this, the tester typically measures these parameters for control purposes.

Additionally, the values of these parameters (along with battery temperature and other variables)[f] must be measured and recorded for later calculation of the parameters of interest, such as those described in Section 1.4.2. This can be done using an external data acquisition system, but most battery test stations at the INEEL also provide this data measurement and logging function. The advantage of this integrated control and measurement approach is that the recorded data can be easily synchronized to the test sequence. Most testers report the test program step as part of the recorded data, and data acquisition section describes a generic "battery test station." Specific information about the test systems treated in this report is presented in subsections below.

A battery test station (often referred to as a *tester*) is a device that applies controlled conditions to the terminals of a battery under test and measures the resulting battery response. Figure 1 is a simple diagram of such a test station, showing its connections to the battery to be tested. All the batteries of interest for hybrid vehicle use are rechargeable, and vehicle operation involves alternating periods of discharge and charge. Thus, a tester is required to act both as a controlled load under battery discharge conditions and a controlled source of power and energy under battery charge conditions, potentially with sampling intervals can usually be varied for each step. Most test stations acquire data at a (relatively high) fixed rate for control purposes and record these data at programmable (typically lower) rates specified by the user.

As indicated in Figure 1, battery current is typically measured using a dc shunt in series with the device under test (though the shunt is often internal to the test station and not accessible for outside measurements). Voltage is preferably measured directly at the battery terminals to avoid errors from line losses at the high currents required.

1.5.1 The Specific Types of Testers Involved in the Study

This study characterizes the measurement uncertainty of test stations built by four manufacturers for the INEEL. Some of these testers (Energy Systems and Bitrode) were custom-built for the INEEL, while others are off-the-shelf equipment representative of the manufacturer's products at the time they were built. Table 1 lists the number and type of testers (for each manufacturer) currently in use at the INEEL, with their full-scale voltage and current measurement and control ranges. Because of the wide variety of operating ranges for the various Maccor testers, only a representative example is treated in this report. In general, the various Maccor models have similar specifications, and the design of their measurement systems is largely unchanged over the several years separating the oldest and newest models.

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f. For multicell batteries, individual cell or module voltages and temperatures are commonly recorded in addition to the overall measurements.

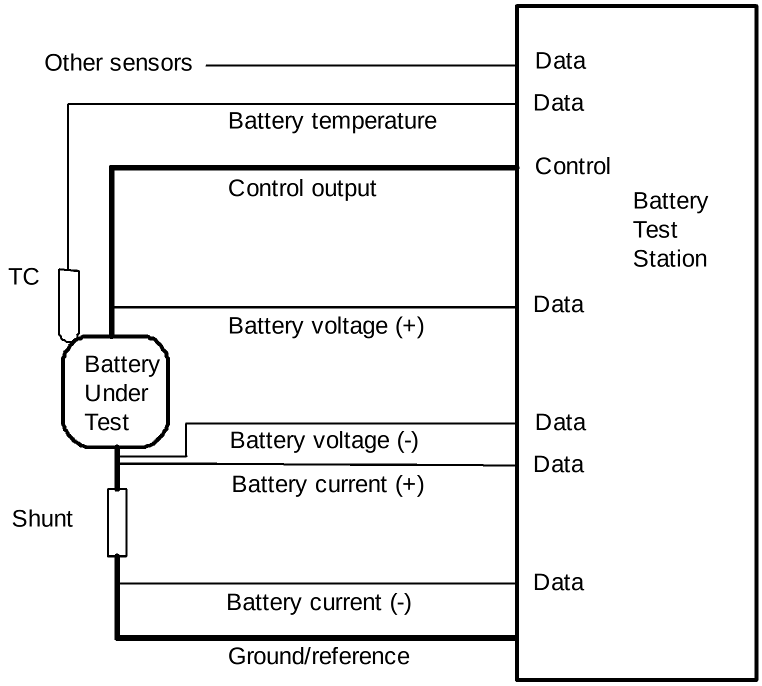

Figure 1. Generic battery test station.

Table 1. INEEL tester types and ranges (as of January 2001).

Tester Manufacturer	Number of Tester Units	Number of Test Channels[a]	Full-Scale Voltage (V)	Full-Scale Current (A)
AeroVironment Model ABC-150	2	2	530	500 (parallel) 250 (independent)
Energy Systems	3	1	500	500[b]
Bitrode	4	1	100	500
Maccor 1	1	5	20	12.5
		2	100	5
		1	100	50
Maccor 2	1	16	20	12.5
		8	5	50
Maccor 3	1	8	5	5
Maccor 4	1	8	5	0.5[c]
Maccor 5	1	8	5	100
Maccor 6, 7	2	7 or 8	5	250
Maccor 8, 9, 10	3	24	10	12.5
Maccor 11	1	8	30	100
Maccor 12	1	4	65	250
Maccor 13	1	8	5	250

a. This corresponds to the number of devices that can be tested simultaneously and independently with a given piece of equipment. It has no relationship to the number of measurement channels provided for each device under test.

b. Energy Systems 1 is limited to ~437 A, but with a 500-A measurement range. All three units can be reconfigured for several (lower) voltage and current ranges.

c. Full-scale range can be adjusted down.

1.5.2 Manufacturers' Claims of Accuracy for the Testers

1.5.2.1 Maccor Testers.

Measured parameters:	Voltage, current, temperature
Resolution of digitizer:	Sixteen bits
Sample rate: adjustable:	One hundred samples per second (sps) maximum
Anti-aliasing filter:	Temperature, first order, 1.41 Hz Voltage and current, none
Accuracy:	0.5°C repeatability, 2.5°C algorithm (original, software now modified to <1°C) 0.02% of full-scale repeatability for voltage and current
Measurement channels:	Model dependent (typically voltage and current with auxiliary channels assignable for temperature and voltage)
Measurement range:	Temperature range is variable, depending on test needs Voltage and current are model dependent (see Table 1)
Calibration:	By procedure, with external reference and measurement

1.5.2.2 Energy Systems Testers.

Measured parameters:	Voltage, current, temperature
Resolution of digitizer:	Twelve bits over +/- 5 Vdc
Sample rate: adjustable:	Ten sps maximum
Anti-aliasing filter:	Temperature, voltage, current: third order, 4.0 Hz
Accuracy:	0.5°C repeatability, high-order polynomial algorithm 0.025% of full-scale repeatability for voltage and current
Measurement channels:	Three primary (voltage, temperature, current) and 59 universal (voltage, temperature or current)
Measurement range:	Temperature dependent on signal conditioning module, but most often 0 to 900°C Voltage, current primary channel: 0 to 500 Vdc, 0 to 500 A (see Table 1). Auxiliary Channels +/-5 Vdc, (including current via external shunt or temperature via various signal conditioning modules)
Calibration:	By procedure, with external reference and measurement

1.5.2.3 Bitrode Testers.

Measured parameters:	Voltage, current, temperature
Resolution of digitizer:	Twelve bits
Sample rate: adjustable:	Ten sps maximum
Anti-aliasing filter:	Unknown
Accuracy:	0.5°C repeatability, algorithm unknown 0.01 V dc, 0.1 A repeatability for main voltage and current 0.01 V dc for cell voltage 0.001 V dc for auxiliary channels
Measurement channels:	Main voltage and current, cell voltage, auxiliary voltage, temperature
Measurement range:	Temperature: -4 to 200°C Voltage, current main channel: 0 to 100 Vdc, 0 to 500 A Cell voltage +/-25 Vdc Auxiliary channels: 0 to 5 Vdc
Calibration:	By procedure, with external reference and measurement

1.5.2.4 AeroVironment Testers.

Measured parameters:	Voltage, current
Resolution of digitizer:	Twelve bits
Sample rate: adjustable:	Four sps maximum
Anti-aliasing filter:	Unknown
Accuracy:	Voltage: +/-250 mV Current (independent or differential): +/-200 mA or 0.25% of reading (whichever is greater) Current (parallel): +/-350 mA or 0.5% of reading (whichever is greater)
Measurement channels:	Voltage, current
Measurement range:	Voltage, current (independent mode): 8 to 420 Vdc, +/-265 Adc Voltage, current (parallel mode): 8 to 420 Vdc, +/-530 Adc Voltage, current (differential mode): +/-420 Vdc, +/-265 Adc
Calibration:	Calibration check only by procedure, with battery-powered DVM and external shunt for current

2. UNCERTAINTY RELATIONSHIPS FOR MEASURED PARAMETERS

This section examines the theory that determines the error or uncertainty in sensing physical parameters that are electronically measured and processed by a digital data acquisition system for battery testing.

2.1 Basic Relationships

For the battery test systems described in Section 1, the typical measured parameters are voltage, current, and temperature. The evaluation approach is the same for any of these parameters, with minor exceptions. Consider the typical parameter denoted by *P*. Figure 2 shows a block diagram for the measurement channel of parameter *P*.

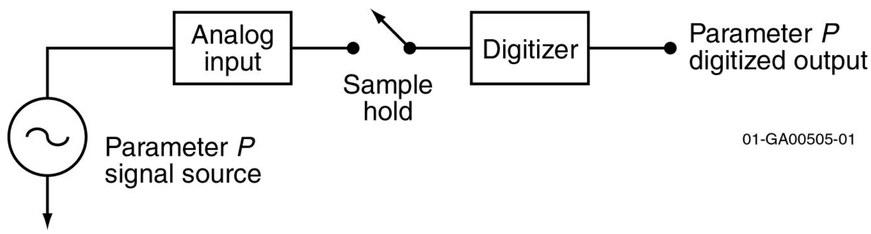

Figure 2. Diagram for typical measured parameter data acquisition.

Parameter *P* digitized output is what the data system receives. That is not necessarily the final result. In most cases, the data system processes the data with some software algorithm to perform engineering units conversion. For example, with a digitizer of N bits, the digitized output could be a binary count ranging from 0 to $\left(2^N - 1\right)$, and the desired output in engineering units might be 0 to 500 Vdc. In this case, the algorithm is likely a simple linear relationship. In another case, such as temperature, the algorithm could be very complex, or perhaps it might be a look-up table. The uncertainty evaluation must consider all sources of error, including data processing. When tests to evaluate uncertainty are run, the test configuration basically implements Figure 2 with the signal source replaced by a calibration source. The total error, $\%Perr_{TOT}$ for measured parameter *P*, including the data processing algorithm, and assuming that error sources are statistically independent and given as a percentage of full scale,[g] is described by

$$\%Perr_{TOT} =$$

$$\sqrt{\left(\%err_{CAL}\right)^2 + \left(\%err_{TD}\right)^2 + \left(\%err_{FE}\right)^2 + \left(\%err_{TJ}\right)^2 + \left(\%err_{QUAN}\right)^2 + \left(\%err_{ALIAS}\right)^2 + \left(\%err_{ALGOR}\right)^2} \quad (1)$$

where

$\%err_{CAL}$ = error associated with the calibration source and process

$\%err_{TD}$ = error associated with the transducer (e.g., thermocouple or current shunt)

$\%err_{FE}$ = error associated with analog front-end electronics

g. Commonly abbreviated as % *FS*.

$\%err_{TJ}$ = error associated with time jitter

$\%err_{QUAN}$ = error associated with quantization, e.g.,

$$\%err_{QUAN} = 100 \times \frac{1}{\left(2^N - 1\right)}, \ N = No.\ of\ bits$$

$\%err_{ALIAS}$ = error associated with signal dynamics of sampled data, called aliasing

$\%err_{ALGOR}$ = error associated with algorithm of engineering units conversion.

We define the equipment error, the error without calibration uncertainty or transducer uncertainty, as

$$\%Peq_{TOT} = \sqrt{(\%err_{FE})^2 + (\%err_{TJ})^2 + (\%err_{QUAN})^2 + (\%err_{ALIAS})^2 + (\%err_{ALGOR})^2} \quad (2)$$

It is possible to estimate some of the terms in Equation (1). For example, if the number of bits is known, the quantization error, $\%err_{QUAN}$, is easily obtained as above. For the effect of time jitter, in the worst case one could assume that the signal is at the maximum rate of change at the instant of sample. Then, the time jitter times that maximum rate of change, referenced to full scale and expressed as a percent, is the worst-case error. However, with the time jitter a small fraction of the sample period, and for the applications of this system being limited to signals that have frequency content close to dc, the assumption of neglecting this error is considered valid. The potential for aliasing errors is treated in some detail in Section 2.2, and it will be examined further in later equipment-specific volumes of this report. In most cases, it will be neglected on the assumptions that (a) test excitation and signal spectra will be kept very near dc, and (b) the devices under test and test setups will minimize external noise pickup. The engineering units conversion error, if knowledge of the algorithm is available, can be estimated. If it is a simple linear relationship, it can be included in the calibration error. Additionally, the calibration error is a combination of offset and sensitivity errors that are here considered lumped into the maximum error that could occur at full scale. Equation (1) becomes Equation (3) with these latter three error sources removed or combined:

$$\%Perr_{TOT} = \sqrt{(\%err_{CAL})^2 + (\%err_{FE})^2 + (\%err_{QUAN})^2 + (\%err_{TD})^2} \quad (3)$$

The equipment manufacturer may provide a specification of the equipment error, $\%Peq_{TOT}$. If such information is available, then Equation (4) applies:

$$\%Perr_{TOT} = \sqrt{(\%err_{CAL})^2 + (\%Peq_{TOT})^2 + (\%err_{TD})^2} \quad (4)$$

2.2 Theory of Sampled Data Aliasing Analysis

This section explains in depth the aliasing error caused by sampling a time varying signal at an insufficiently high sample frequency relative to the spectral content of the signal. The section also develops the necessary relations required to quantify the aliasing error, $\%err_{ALIAS}$.

Transitioning signals from continuous signal space to discrete signal space via sampling to enable digital data processing can lead to the possibility of aliasing. An aliased signal contains a special type of noise that corrupts the signal like any other noise. The following development will illustrate and quantify aliasing noise. Consider a time varying signal, $f(t)$, which has a Fourier transform, where

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \qquad (5)$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \, .$$

Equation (5) is the standard Fourier transform pair. Consider another function of time, $S(t)$, which is a periodic train of impulses that occurs at a time step of Δt, where

$$S(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta t) \, . \qquad (6)$$

$S(t)$ is called the sampling function. Consider the product of $f(t)$ and $S(t)$:

$$h(t) = S(t) f(t) = \sum_{n=-\infty}^{\infty} f(t) \delta(t - n\Delta t)$$

$$h(t) = \sum_{n=-\infty}^{\infty} f(n\Delta t) \delta(t - n\Delta t) \, . \qquad (7)$$

Now consider the Fourier transform of Equation (7):

$$H(\omega) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(n\Delta t) \delta(t - n\Delta t) e^{-j\omega t} dt$$

$$H(\omega) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(n\Delta t) \delta(t - n\Delta t) e^{-j\omega t} dt$$

$$H(\omega) = \sum_{n=-\infty}^{\infty} f(n\Delta t) e^{-j\omega n \Delta t}$$

$$G(\omega) \equiv H(\omega) \Delta t = \sum_{n=-\infty}^{\infty} f(n\Delta t) e^{-j\omega n \Delta t} \Delta t \, . \qquad (8)$$

Equation (8) is comparable to Equation (5). In fact, as the $Limit \; \Delta t \to 0$, Equation (8) becomes Equation (5). Now, consider Equations (5) and (8) evaluated at $\omega = 0$:

$$G(0) = \sum_{n=-\infty}^{\infty} f(n\Delta t) \Delta t \, , \; F(0) = \int_{-\infty}^{\infty} f(t) dt$$

As the $Limit\ \Delta t \to 0$, $G(0) = F(0)$. Now, consider $\omega = \frac{2\pi}{\Delta t}$. Equation (5) becomes

$$F\left(\frac{2\pi}{\Delta t}\right) = \int_{-\infty}^{\infty} f(t) e^{-j\frac{2\pi}{\Delta t}t} dt$$

This is simply the Fourier transform evaluated at a specific frequency. Now, consider Equation (8) at that frequency:

$$G\left(\frac{2\pi}{\Delta t}\right) = \sum_{n=-\infty}^{\infty} f(n\Delta t) e^{-j\frac{2\pi}{\Delta t}n\Delta t} \Delta t = \sum_{n=-\infty}^{\infty} f(n\Delta t) \underbrace{e^{-jn2\pi}}_{1} \Delta t = G(0) .$$

The response is back to dc, and it appears periodic. Consider $G(\Delta\omega)\ and\ G\left(\frac{2\pi}{\Delta t} + \Delta\omega\right)$:

$$G(\Delta\omega) = \sum_{n=-\infty}^{\infty} f(n\Delta t) e^{-j\Delta\omega n\Delta t} \Delta t$$

$$G\left(\frac{2\pi}{\Delta t} + \Delta\omega\right) = \sum_{n=-\infty}^{\infty} f(n\Delta t) e^{-j\left(\frac{2\pi}{\Delta t} + \Delta\omega\right)n\Delta t} \Delta t = \sum_{n=-\infty}^{\infty} f(n\Delta t) \underbrace{e^{-j(2\pi)n}}_{1} e^{-j(\Delta\omega)n\Delta t} \Delta t = G(\Delta\omega)$$

It is in fact periodic. Now consider $G(-\Delta\omega)\ and\ G\left(\frac{2\pi}{\Delta t} - \Delta\omega\right)$:

$$G(-\Delta\omega) = \sum_{n=0}^{\infty} f(n\Delta t) e^{j\Delta\omega n\Delta t} \Delta t$$

$$G\left(\frac{2\pi}{\Delta t} - \Delta\omega\right) = \sum_{n=-\infty}^{\infty} f(n\Delta t) e^{-j\left(\frac{2\pi}{\Delta t} - \Delta\omega\right)n\Delta t} \Delta t = \sum_{n=-\infty}^{\infty} f(n\Delta t) \underbrace{e^{-j(2\pi)n}}_{1} e^{j(\Delta\omega)n\Delta t} \Delta t = G(-\Delta\omega) .$$

This means that the complete spectrum must consider negative frequency, and it is periodic at the sample frequency. Figures 3, 4, 5, and 6 illustrate the nature of the waveforms of the sampling process.

As seen in the spectrum of the sampled *f*(*t*), a perfect low-pass filter can recover the "fundamental," which has the identical shape as the pure spectrum of *f*(*t*). Observe the frequency halfway between dc and the sample frequency $\left(\omega_n = \frac{\pi}{\Delta t}\right)$. If there is no spectral content beyond this frequency, a perfect low-pass filter will capture 100% of the spectral content of the original signal spectrum. Thus, with an inverse Fourier transform operation, *f*(*t*) can be completely recovered. Now, if the signal has spectral content beyond ω_n (the Nyquist frequency), then a low-pass filter cannot recover the original spectrum but will

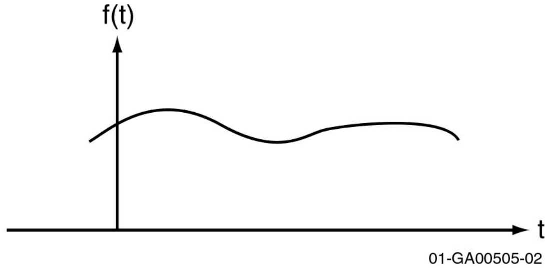

Figure 3. Time signal f(t).

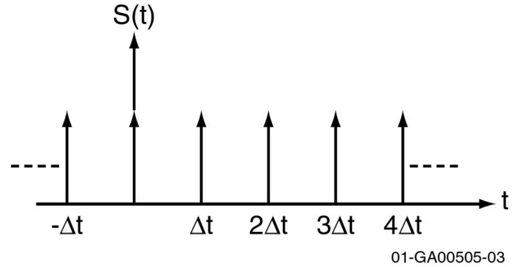

Figure 4. Sampling function S(t).

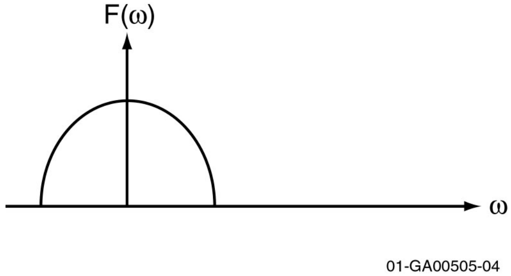

Figure 5. Frequency spectrum of f(t).

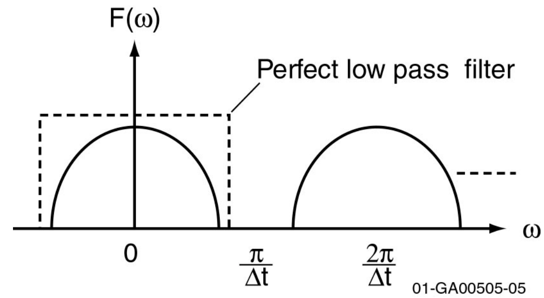

Figure 6. Spectrum of sampled signal.

instead recover a spectrum that consists of the original corrupted with some of the tail from the first harmonic spectrum. This corruption is called aliasing noise. Figure 7 illustrates this effect.

The effect of this is introduction of noise into the recovered signal. The effect can be quantified. The convention for noise analysis is signal power compared to noise power. *Signal power* is power in the signal up to the Nyquist frequency. *Noise power* is the power in the overlapping tail or the power in the signal power spectrum from the Nyquist frequency to ∞ . The power spectrum is the signal spectrum times its complex conjugate (replace every j by -j) denoted by F*. The units of signal spectrum are assumed as volts/Hz. If a 1-ohm load is also assumed, the resulting power spectrum has units of watts/Hz^2. Additionally, the spectrum is assumed symmetrical with negative frequency, and thus the integration to obtain power need only be performed over positive frequency.

$$\text{Signal Power} \equiv \int_0^{\omega_n} F(\omega)F^*(\omega)d\omega \tag{9}$$

$$\text{Noise Power} \equiv \int_{\omega_n}^{\infty} F(\omega)F^*(\omega)d\omega \tag{10}$$

Note that the relationship between signal power (or energy) in the time domain and the frequency domain is defined by Parseval's relationship:

$$\int_{-\infty}^{\infty} \left| f(t)^2 \right| dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| F(\omega) \right|^2 d\omega \ .$$

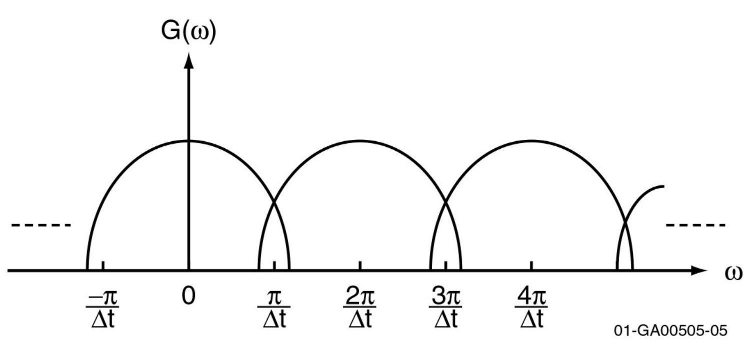

Figure 7. Fundamental spectrum contaminated by the first harmonic.

The aliasing error, $\sigma_{ALIASING}$ is given by

$$\sigma_{ALIASING} = \sqrt{\frac{noise\ power}{signal\ power}}, \quad \%err_{ALIAS} = 100\,\sigma_{\mathrm{ALIASING}}\,. \tag{11}$$

To perform an analysis of aliasing, the approach is as follows. A typical data acquisition system has in its signal path an antialiasing low-pass filter. Figure 8 shows a block diagram of the typical front end of a sampled data system. If one assumes that the signal is white noise, the signal spectrum is then the transfer function (in terms of $j\omega$) of the antialiasing filter, $H(\omega)$. Equations (9), (10), and (11) are applied and the error computed. Because the power spectrum of most low-pass filters will cause Equations (9) and (10) to become a very messy integration, a numerical approach should be pursued with software such as Matlab or custom code. In the numerical integration, it is not necessary or practical to integrate to infinity. Instead, the upper limit of integration can be set to $10\omega_n$ and then repeated using $20\omega_n$. If there is no significant change, then either value is adequate as the upper limit. If there is a change, the upper limit can be increased until there is no significant change.

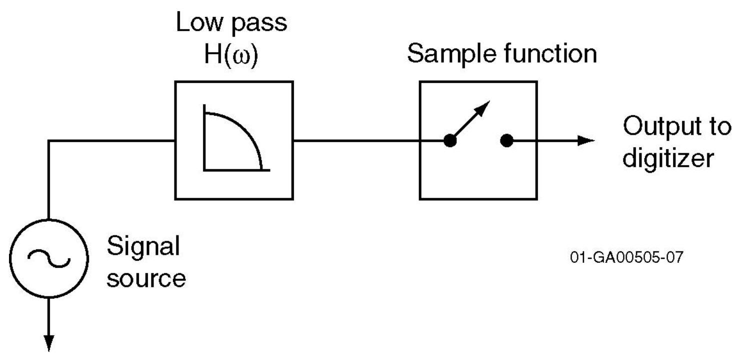

Figure 8. Diagram of a typical data acquisition system.

2.3 Analytical Relationships for Evaluation Testing

In evaluating system uncertainty, testing is an important tool, as it enables a reality check for the system under evaluation. The testing, if it is thorough, representative, and with statistically adequate sample sizes, can be the sole basis for the system uncertainty. More often, testing will be part of the process of the evaluation. It will help establish typical system uncertainty performance. The complete uncertainty evaluation will include operational experience, the system manufacturer's data, and testing. The following analysis allows the system evaluator to obtain the needed degree of testing for the uncertainty evaluation.

Consider a test where large steady-state time records of parameter P data, spread over the parameter's range, are collected for multiple measurement channels and statistically processed. The number of channels used and the number of steps taken over the parameter range are as required to obtain the desired confidence level of the statistical results. The steady-state signal source for the test eliminates the aliasing error term. Additionally, assume that during the calibration process for this test, an end-to-end calibration is performed such that uncertainty of a transducer is eliminated. Consider that the time records of data at specific range levels are processed to find the mean [Equation (12)] and the standard deviation [Equation (13)]. These relations will apply at a specific range level.

$$\bar{P}_{jk} = \frac{1}{N}\sum_{i=1}^{N} P_{i\,jk} \tag{12}$$

$$\sigma_{Pjk} = \sqrt{\frac{1}{N-1}\sum_{\mathrm{i}=1}^{N}\left(P_{i\,jk} - \bar{P}_{jk}\right)^2} \tag{13}$$

where

N = the total number of time samples

i = the time sample index integer

j = the index integer for a specific channel number

k = the index integer for range

FS = the full-scale value of P for the measurement process under evaluation.

The results of the application of Equations (12) and (13) are functions of range level and channel number. Additionally, Equation (14) defines an error for the averaged measured parameter $\overline{P}_{jk}$. It also is a function of range level and channel number.

$$\%P_{ERR\,jk} = 100 \times \frac{Reference_k - \bar{P}_{jk}}{FS} \;. \tag{14}$$

Reference$_k$ in Equation (14) may be a calibration standard or set point for that specific range level. The parameters given by Equations (13) and (14) are averaged over the various specific range levels and the number of channels. Parameter $\%P_{ERR\,jk}$ is averaged as stated and illustrated by Equation (15). The result is $\%\overline{P}_{ERR}$, the average offset error as a percentage of full scale:

$$\%\overline{P}_{ERR} = \frac{1}{M}\sum_{j\#Chan}^{M}\frac{1}{L}\sum_{\substack{k\#Range\\Levels}}^{L}\%P_{ERR\,jk} \tag{15}$$

where

M = number of channels tested

L = number of range levels tested for each channel.

Note that this kind of error is systematic and as such could possibly be corrected. However, we will assume that this is not practical. Additionally, we will assume that this error is statistically independent of any other error sources.

The standard deviation σ_{Pjk} is also averaged as stated and illustrated by

$$\%Perr_{STD} = \frac{100}{FS}\frac{1}{M}\sum_{j\#Chan}^{M}\frac{1}{L}\sum_{\substack{k\#Range\\Levels}}^{L}\sigma_{Pjk} \quad . \tag{16}$$

The error parameters $\%\bar{P}_{ERR}$ *and* $\%Perr_{STD}$ derived from testing are assumed statistically independent of each other. The resulting estimate of the equipment error based upon testing is given by

$$\%Peq_{TOT} = \sqrt{(\%Perr_{STD})^2 + (\%\bar{P}_{ERR})^2 - (\%err_{CAL})^2} \tag{17}$$

where

$\%Perr_{CAL}$ = the calibration process error for this testing; as a conservative assumption this is set to zero

$\%Peq_{TOT}$ = the uncertainty component for the equipment error for this measurement.

The uncertainty $\%Perr_{TOT}$ of a measurement of parameter P, using the testing results, is given by

$$\%Perr_{TOT} = \sqrt{(\%Peq_{TOT})^2 + (\%err_{CAL})^2 + (\%err_{TD})^2} \tag{18}$$

where

$\%Perr_{TOT}$ = uncertainty of parameter P as a percentage of *FS*

$\%err_{CAL}$ = average uncertainty of the calibration process performed for the measurement (as a percentage of *FS*).

$\%err_{TD}$ = uncertainty of the transducer for parameter P as a percentage of *FS*.

Observe that Equation (18) is identical to Equation (4) and forms the basis of comparing the uncertainty evaluation obtained via testing with the manufacturer's specified uncertainty. If testing was not done, and the equipment supplier provides a specification for $\%Peq_{TOT}$, Equation (18) can still be used. If the equipment supplier has not provided a specified uncertainty, the general relationship for uncertainty in Equation (2) relates the uncertainty to typical errors of a data acquisition system, which must be obtained by analysis or estimation. Equation (2) can be reduced to Equation (19) by removing time jitter and aliasing error sources, which are neglected in further analysis:

$$\%Peq_{TOT} = \sqrt{(\%err_{FE})^2 + (\%err_{QUAN})^2 + (\%err_{ALGOR})^2} \tag{19}$$

where

$\%err_{FE}$ = error associated with analog front-end electronics

$\%err_{QUAN}$ = the quantization error expressed as a percentage of *FS*, e.g.,

$$\%err_{QUAN} = 100 \times \frac{1}{\left(2^N - 1\right)}, \quad N = No.\ of\ Bits$$

$\%err_{ALGOR}$ = error associated with algorithm of engineering units conversion.

2.4 Application to System Measurements

2.4.1 Temperature Measurement

An evaluation of equipment uncertainty for temperature measurement based on testing will use Equation (17) restated in terms of temperature:

$$\%Teq_{TOT} = \sqrt{\left(\%Terr_{STD}\right)^2 + \left(\%\overline{T_{ERR}}\right)^2 - \left(\%err_{TCAL}\right)^2} \tag{20}$$

where

$\%Teq_{TOT}$ = uncertainty component of the temperature measurement channel as a percentage of full scale

$\%Terr_{STD}$ = average standard deviation of the test data set of parameter T as a percentage of *FS* [obtained from a large time record over range and multiple measurement channels using Equations (13) and (16)]

$\%\overline{T_{ERR}}$ = average error of the test data set of parameter T as a percentage of *FS* [obtained from a large time record over range and multiple measurement channels using Equations (12), (14), and (15)]

$\%err_{TCAL}$ = average uncertainty (as a conservative assumption set to zero) of the calibration for the testing done to obtain parameters $\%Terr_{STD}$ and $\%\overline{T_{ERR}}$.

Equation (19) is restated here in the form of temperature for applications where neither testing nor manufacturer uncertainty data are available:

$$\%Teq_{TOT} = \sqrt{\left(\%Terr_{FE}\right)^2 + \left(\%Terr_{QUAN}\right)^2 + \left(\%Terr_{ALGOR}\right)^2} \tag{21}$$

where

$\%Teq_{FE}$ = error associated with analog front-end electronics

$\%Teq_{QUAN}$ = quantization error expressed as a percentage of *FS*

$\%Teq_{ALGOR}$ = error associated with algorithm of engineering units conversion.

In Equation (21), observe that for temperature the error $\%Terr_{ALGOR}$ is not usually neglected without some consideration, as the relationship between the sensor digitized voltage and the actual temperature being sensed is typically not a linear relationship. A high-order polynomial or a look-up table usually best approximates this relationship. For example, with 0 to 100°C full scale and a simple linear approximation, the error for a type T thermocouple sensor can be as much as 2.5°C over that full-scale range. When the equipment manufacturer implements an algorithm based on a high-resolution look-up table or a high-order polynomial, this error is often neglected.

Temperature testing is typically performed to obtain the parameters of Equation (20) using multiple measurement channels. An end-to-end calibration is performed on these channels to eliminate the transducer error. A recommended approach consists of placing the actual channel sensors (normally thermocouples for INEEL testers) into a reference temperature source, which is able to provide any calibration temperatures over the desired range for the testers in question. Adequate equilibration time must be allowed before the calibration measurement is taken. The calibration uncertainty using this method is that of the temperature reference. The test itself would use the reference temperature source, which is applied simultaneously to the different channel thermocouples. For a full-scale range of 0 to 100°C, the temperature of the reference could be stepped from 0 to 100°C to obtain data over the full-scale range. After the equilibration period, approximately 1000 data points should be acquired for each channel at each step. Then, for each channel and at each step, the mean and standard deviation are computed with Equations (12) and (13). The average error for each channel and each temperature step is obtained with Equation (14). In Equation (14), the reference is considered the set point of the reference temperature source. The overall average error $\%\overline{T}_{ERR}$ and average standard deviation $\%Terr_{STD}$ are computed with Equations (15) and (16). The uncertainty specification of the reference temperature source is also the parameter $\%\,err_{TCAL}$ if this source is used for the calibration. Thus, all the information for Equation (20) is available, and the equipment component for temperature uncertainty, $\%Teq_{TOT}$, can be obtained. To obtain the temperature measurement uncertainty for a typical application, Equation (18) [restated here in terms of temperature as Equation (22)] is applied:

$$\%err_T = \sqrt{(\%Teq_{TOT})^2 + (\%err_{TCAL})} \quad . \tag{22}$$

Observe that the transducer error from the sensor, $\%err_{TD}$, appears to be missing from Equation (22). It is included with the calibration error, as follows. For temperature measurement, two methods of channel calibration are possible.

1. Use of a simulated thermocouple source (such as a hand-held calibrator), where calibration errors from both the simulated source and the thermocouple result in Equation (23):

$$\%err_{TCAL} = \frac{100}{FS}\sqrt{(STc)^2 + (Tc)^2} \tag{23}$$

where

STc = uncertainty of the simulated thermocouple source in °C

Tc = uncertainty of the actual thermocouple

FS = full-scale temperature range in °C.

2. An end-to-end calibration using a reference temperature source (such as a reference oven), with the only error from the reference, which yields

$$\%err_{TCAL} = \frac{100}{FS}(RT) \tag{24}$$

where

RT = range error in °C for the reference temperature source

FS = full-scale temperature range in °C.

The calibration of Case 1 is typically more convenient and has been most often used. Case 2, which is less convenient, is available for situations requiring higher accuracy.

2.4.2 Current Measurement

Equation (17), as applied to the current measurement, becomes

$$\%Ieq_{TOT} = \sqrt{(\%Ierr_{STD})^2 + \left(\%\overline{I_{ERR}}\right)^2 - (\%err_{ICAL})^2} \tag{25}$$

where

$\%Ieq_{TOT}$ = uncertainty component of the current measurement channel as a percentage of full scale

$\%Ierr_{STD}$ = average standard deviation of the test data set of parameter I as a percentage of FS [obtained from a large record of time over range and multiple measurement channels using Equations (13) and (16)]

$\%\overline{I_{ERR}}$ = average error of the test data set of parameter I as a percentage of FS [obtained from a large record of time over range and multiple measurement channels using Equations (12), (14), and (15)]

$\%err_{ICAL}$ = average uncertainty of the calibration and reference for the testing to obtain parameters $\%Ierr_{STD}$ and $\%\overline{I}_{ERR}$ (typically set to zero as a conservative assumption).

In the various battery test systems to be evaluated, the test system provides and measures the current. The provided current is a current source and is precisely regulated. Additionally, the current shunt used to measure the current is typically part of the tester, and its error is lumped with the equipment error. In performing current testing to obtain the parameters of Equations (12), (13), (14), (15), and (16), multiple measurement channels should be used where practical. A calibration can be performed using a high-precision digital voltmeter (DVM) and an external current shunt. The reference in Equation (14) is the external shunt current measurement expressed as a percentage of full scale. For current calibration uncertainty, errors are from the shunt and from the precision DVM monitoring the shunt. For the DVM, the error typically depends on the range used. The resultant $\%err_{ICAL}$ is given by

$$\%err_{ICAL} = 100\sqrt{(err_{SHUNT})^2 + \left(\frac{DVMerr}{FS\ Cal.\ Shunt\ Voltage}\right)^2} \tag{26}$$

where

err_{SHUNT} = shunt error expressed as a decimal fraction (not %)

$DVMerr$ = DVM error expressed as voltage (not % or a decimal fraction).

To obtain the current measurement for a typical application, Equation (18) [restated here as Equation (27) with the term $\%err_{TD}$ deleted], is applied:

$$\%err_I = \sqrt{(\%Ieq_{TOT})^2 + (\%err_{ICAL})^2}\ . \tag{27}$$

Channel calibration typically uses an external shunt for current measurement. The DVM and the calibration shunt error are parts of the calibration error in Equation (26). The calibration error term $\%err_{ICAL}$ in Equation (27) could be different from the value applied in Equation (25) (due to the measurement range in use, for example) and is not set equal to zero. Thus, this term must be calculated using Equation (26). The other term in Equation (27), $\%Ieq_{TOT}$, is obtained from Equation (25).

2.4.3 Voltage Measurement

Equation (17), as applied to the voltage measurement, becomes

$$\%Veq_{TOT} = \sqrt{(\%Verr_{STD})^2 + \left(\%\overline{V}_{ERR}\right)^2 - (\%err_{VCAL})^2} \tag{28}$$

where

$\%Veq_{TOT}$ = uncertainty component of the voltage measurement channel as a percentage of *FS*

$\%Verr_{STD}$ = standard deviation of the test data set of parameter *V* as a percentage of *FS* [a large record of time over range and multiple measurement channels using Equations (13) and (16)]

$\%\overline{V}_{ERR}$ = average error of the test data set of parameter *V* as a percentage of *FS* [a large record of time over range and multiple measurement channels using Equations (12), (14), and (15)]

$\%err_{VCAL}$ = average uncertainty of the calibration and reference for the testing to obtain parameters $\%Verr_{STD}$ and $\%\overline{V}_{ERR}$ (typically set to zero as a conservative assumption).

Various battery test systems may measure voltages with many channels. The front-end electronics inherently measures voltage, and a separate transducer is not used. The transducer error term $\%err_{TD}$ is thus not necessary. Typical voltage testing performed to obtain the parameters of Equations (12), (13), (14), (15), and (16) would use several measurement channels. A calibration is typically performed using a precision DVM and a precision voltage source. The reference in Equation (14) is either the voltage source set point or the DVM reading. For voltage calibration, the uncertainty error is from the DVM and the voltage reference. The resultant $\%\mathrm{err}_{\mathrm{VCAL}}$ is given by

$$\%err_{VCAL} = 100\sqrt{\left(\frac{DVMerr}{Full - Scale\,Voltage}\right)^2 + \left(\frac{VSerr}{Full - Scale\,Voltage}\right)^2} \tag{29}$$

where

$DVMerr$	=	DVM error expressed as a voltage
$VSerr$	=	voltage reference error expressed as a voltage. (If the DVM reading is used as a reference, this component contains only the voltage stability error.)

To obtain the voltage measurement uncertainty for a typical application, Equation (18) [restated here as Equation (30) with the term $\%err_{TD}$ deleted] is applied:

$$\%err_V = \sqrt{(\%Veq_{TOT})^2 + (\%err_{VCAL})^2} \quad . \tag{30}$$

Typically for voltage measurement, the channel calibration is performed using a precision *DVM* and a voltage reference. The calibration error term, $\%err_{VCAL}$, in Equation (30) could be different than that applied in Equation (28), and thus this term must be calculated using Equation (29). The other term in Equation (30), $\%Veq_{TOT}$, is obtained from Equation (28).

3. UNCERTAINTY RELATIONSHIPS FOR DERIVED PARAMETERS

3.1 Theoretical Background

3.1.1 General

The derived parameters used in battery tests and evaluations for the PNGV and Advanced Technology Development Programs are based upon the obtained measured parameters. The uncertainty of the derived parameters is thus a function of the uncertainty of the associated measured parameters. This section states the general uncertainty expressions for derived parameters and then develops specific expressions for each of the various derived parameters.

We start with the Taylor Series expansion of a function F, truncated to include only the first derivative terms for a multivariable function:

$$F(P_1 + \Delta P_1, P_2 + \Delta P_2, P_3 + \Delta P_3 .....) = F(P_1, P_2, ...) + \frac{\partial F}{\partial P_1}\Delta P_1 + \frac{\partial F}{\partial P_2}\Delta P_2 + \frac{\partial F}{\partial P_3}\Delta P_3 + ... \quad (31)$$

From Equation (31), $\Delta F(P_1, P_2, ...) = \frac{\partial F}{\partial P_1}\Delta P_1 + \frac{\partial F}{\partial P_2}\Delta P_2 + \frac{\partial F}{\partial P_3}\Delta P_3 + ...$

$$\%F = 100\frac{\Delta F(P_1, P_2, ...)}{F_{FS}} = \frac{100}{F_{FS}}\left(\frac{\partial F}{\partial P_1}\Delta P_1 + \frac{\partial F}{\partial P_2}\Delta P_2 + \frac{\partial F}{\partial P_3}\Delta P_3 + ...\right)$$

where

ΔP_i	=	$P_{iFS}\frac{errP_i\%}{100}$
$errP_i\%$	=	variation of parameter P_i in percentage of full scale
P_{iFS}	=	full-scale value of parameter P_i.

If variations of parameters of F were not random and we continue to neglect all but the first derivative terms, small changes in F caused by slight variations by any of the parameters P_i would be approximated by

$$\%F = \frac{1}{F_{FS}}\left(\frac{\partial F}{\partial P_1}P_{1FS}errP_1\% + \frac{\partial F}{\partial P_2}P_{2FS}errP_2\% + \frac{\partial F}{\partial P_3}P_{3FS}errP_3\% + ...\right). \quad (32)$$

However, the variation in the parameters P_i of F are assumed to be small, random, and statistically independent. Thus, each error contribution, $\Delta\%F_i = \frac{1}{F_{FS}}\left(\frac{\partial F}{\partial P_i}P_{iFS}errP_i\%\right)$, will combine as the root sum square (RSS) of all the errors. Equation (33) is obtained as a result:

$$\%F = \frac{1}{F_{FS}}\sqrt{\sum_i\left(\frac{\partial F}{\partial P_i}P_{iFS}errP_i\%\right)^2}\ . \quad (33)$$

Equation (33) is based on the assumptions of random variations, statistical independence, and small errors.

The error associated with a parameter can come from both calibration and random noise sources. To better understand this, consider the general form of the error factor:

$$\Delta P = P_{OS} + \sigma .$$

P_{OS} represents a constant shift of error at some specific P_{TRUTH}. σ is the standard deviation part of the error that is random or noise. P_{OS} is not simply a constant but is a lumping of both linearity and true offset. If we consider the simple calibration equation

$$y = mx + B$$

where x is the "raw" digitized data and y is the desired engineering units, m becomes the calibration linearity or sensitivity and B is the calibration offset.

P_{OS} represents a lumping of the error part of m along with the error part of B, both referenced to full scale. Throughout this report, the most conservative assumptions have been made with respect to determining the error associated with each parameter.

3.1.2 Difference Functions

Most of the derived parameters are multivariable; either a function of different measured parameters as in the case of Energy or a function of other derived parameters as in the case of Efficiency. Several of the derived parameters contain terms that are differences between the same parameter at different times. The Taylor Series expansion must consider this difference between the same parameter as an additional parameter

$$F = F\left(\ldots,\ \left(P_i(t_1) - P_i(t_2)\right), \ldots\right).$$

Thus the partial derivative of this parameter must also be computed:

$$\frac{\partial F}{\partial\left(P_i(t_1) - P_i(t_2)\right)} .$$

This term would appear in Equation (33) as

$$\left[\frac{\partial F}{\partial\left(P_i(t_1) - P_i(t_2)\right)} \Delta\left(P_i(t_1) - P_i(t_2)\right)\right]^2 .$$

The error factor, $\Delta\left(P_i(t_1) - P_i(t_2)\right)$, is unique to the difference and requires separate consideration. The additional error term for this case can only come from linearity, since any offset error would subtract out when the difference was taken.

A special case arises when any of the $P_i(t_1) - P_i(t_2)$ terms appear individually as well as in a difference term. For this case, we take the conservative approach and apply the total calibration error to the individual instance and also assume this is all linearity error when the term appears as a difference.

3.1.3 Integral Functions

Some of the derived parameters involve time integrals. Consider the general form of a measured variable:

$$V(t) = e_N(t) + V_{OS} + V_{LOS} V_{TRUTH}(t) + V_{TRUTH}(t) \qquad (34)$$

where

$e_N(t)$	=	random error term assumed to have a normal statistical distribution with zero mean and standard deviation σ
V_{OS}	=	error at time of calibration that accounts for zero and sensitivity shifts (assumed fixed at that time)
V_{LOS}	=	error of linearity inherent in the equipment (also assumed fixed)
V_{TRUTH}	=	actual true value of the variable.

In the evaluation of the uncertainty of measured parameters, all errors have generally been combined into the terms $e_N(t)$ and V_{OS}, with V_{LOS} neglected, as it is assumed accounted for by V_{OS}. Thus, Equation (34) becomes

$$V(t) = e_N(t) + V_{OS} + V_{TRUTH}(t) \ . \qquad (35)$$

It is necessary to estimate e_N and/or V_{OS} in many of the relationships to be developed. A usable assumption is generally to let the calibration error be V_{OS} and the standard deviation (derived from tests) be the random term. The calibration error is denoted by $\%err_{VCAL}$ for voltage or $\%err_{ICAL}$ for current. Additionally, if the standard deviation is not available, the equipment error, $\%Ieq_{TOT}$, for current, or $\%Veq_{TOT}$ for voltage, can be used as a conservative estimate.

When Equation (35) is integrated over time, the term $e_N(t)$ will average to zero, but the term V_{OS} will accumulate:

$$\Rightarrow \int V(t)dt = V_{OS} \int dt + \int V_{TRUTH}(t)dt \ . \qquad (36)$$

3.1.4 Integrals of Products

Some of the derived parameters are integrals of products. Again, starting with the Taylor series as applied to an integral function,

$$F = \int P_1 P_2 dt$$

and assuming the independence of P_1 and P_2 allows the partial derivative operator to be moved inside the integral operator,

$$\Delta F = \frac{\partial}{\partial P_1}\left\{\int P_1 P_2 dt\right\}\Delta P_1 + \frac{\partial}{\partial P_2}\left\{\int P_1 P_2 dt\right\}\Delta P_2 = \left\{\int P_2 dt\right\}\Delta P_1 + \left\{\int P_1 dt\right\}\Delta P_2 \quad . \tag{37}$$

This can be put in the form of Equation (33), with $\Delta P_1 = \frac{P_{1FS} errP_1\%}{100}, \Delta P_2 = \frac{P_{2FS} errP_2\%}{100}$, and by assuming errors are random and statistically independent:

$$\%F = \frac{100\Delta F}{F_{FS}} = \frac{1}{F_{FS}}\sqrt{\left(\left\{\int P_2 dt\right\}P_{1FS} errP_1\%\right)^2 + \left(\left\{\int P_1 dt\right\}P_{2FS} errP_2\%\right)^2} \quad . \tag{38}$$

To determine whether Equation (38) is sensible, consider the derived parameter energy, *E*:

$$E = \int VIdt = \int (V_{TRUE}(t) + \Delta V)(I_{TRUE}(t) + \Delta I)dt$$

where ΔV and ΔI are error terms.

The same argument for neglecting the random error term $e_N(t)$ is applied, and $\Delta V = V_{OS}, \Delta I = I_{OS}$.

$$E = \int \left(V_{TRUE}(t) I_{TRUE}(t) + V_{OS} I_{TRUE}(t) + V_{TRUE}(t) I_{OS} + V_{OS} I_{OS}\right)dt$$

$$E = \int V_{TRUE}(t) I_{TRUE}(t)dt + V_{OS}\int I_{TRUE}(t)dt + I_{OS}\int V_{TRUE}(t)dt + I_{OS} V_{OS}\int dt$$

$$\Delta E = \quad V_{OS}\int I_{TRUE}(t)dt + I_{OS}\int V_{TRUE}(t)dt + I_{OS} V_{OS}\int dt \quad . \tag{39}$$

If we ignore the last term in Equation (39), because it is the product of two small numbers, it is similar to Equation (37). Thus, because of the integral effect, the calibration portion of the error is substituted in Equation (37) to obtain uncertainty.

To determine whether the assumptions that allowed the result of Equation (38) are viable, consider that the integral function used to process integrals of products is in fact a summation algorithm processing discrete parameter data, as given by Equation (40).

$$F = \int P_1 P_2 \,\mathrm{dt} = \sum_{i=1}^{N} P_{1i} P_{2i} \Delta t \tag{40}$$

where P_{1i}, P_{2i} are the parameters 1 and 2 at the ith time step Δt .

Additionally, parameters 1 and 2 at the ith time step Δt can be expressed as a true value, with an added error:

$$P_{1i} = P_{1i\,TRUE} + \Delta P_{1i}, \quad \Delta P_{1i} = P_{10} + \sigma_{1i}$$

$$P_{2i} = P_{2i\,TRUE} + \Delta P_{2i}, \quad \Delta P_{2i} = P_{20} + \sigma_{2i} \tag{41}$$

where the error terms, $\Delta P_{1i}, \Delta P_{2i}$, consist of an average offset error, P_{10}, P_{20}, and noise terms, σ_{1i}, σ_{2i}, with a standard deviation of σ_1, σ_2 and zero mean. The relation in Equation (40) is the same form as the discrete cross correlation[h] between parameters P_1, P_2:

$$r_{P_1P_2}(\tau) = \frac{1}{\mathrm{N} - |\tau|} \sum_{i=1}^{N} P_{1i} P_{2i-\tau} \tag{42}$$

where

$r_{P_1P_2}(\tau)$	=	the cross correlation between P_1, P_2
τ	=	the discrete time shift between P_1, P_2
N	=	the number of data points in P_1, P_2.

Now, substitute the error expression of Equation (41) into Equation (40):

$$F = \Delta t \sum_{i=1}^{N} \left(P_{1iTRUE} + P_{10} + \sigma_{1i}\right)\left(P_{2iTRUE} + P_{20} + \sigma_{2i}\right) = F_{TRUE} + \Delta F_{err}$$

$$\Delta F_{\mathrm{ERR}} = \Delta t \sum_{i=1}^{N} \left(P_{1iTRUE} P_{20} + P_{1iTRUE}\sigma_{2i} + P_{2iTRUE} P_{10} + P_{2iTRUE}\sigma_{1i} + P_{10}P_{20} + P_{10}\sigma_{2i} + P_{20}\sigma_{1i} + \sigma_{1i}\sigma_{2i}\right).$$

The last 4 terms, $P_{10}P_{20}, \sigma_{1i}\sigma_{2i}, \mathrm{P}_{10}\sigma_{2i}, \mathrm{P}_{20}\sigma_{1i}$, are either the product of small numbers or because the summation will average to zero. Additionally, the terms $P_{1iTRUE}\sigma_{2i}, P_{2iTRUE}\sigma_{1i}$ will average to zero as the σ_i noise term has zero mean. The final result is

$$\Delta F_{ERR} = \Delta t P_{20} \sum_{i=1}^{N} P_{1\mathrm{i}} + \Delta t P_{10} \sum_{i=1}^{N} P_{2i} \tag{43}$$

where the data sets, P_{1i}, P_{2i}, are used in place of the unknown ideal sets, P_{1iTRUE}, P_{2iTRUE}. Since the error constituents of Equation (43) are statistically independent, Equation (43) must be modified with the RSS operation to become Equation (44):

$$\Delta F_{ERR} = \sqrt{\left(P_{10} \sum_{i=1}^{N} P_{2i}\Delta t\right)^2 + \left(P_{20} \sum_{i=1}^{N} P_{1i}\Delta t\right)^2} \quad . \tag{44}$$

Comparing Equation (44) with Equation (38) (recognizing that Equation (38) is expressed in percent while Equation (44) is in statistical error magnitude), we see that the summation terms are the integrals, and the mean error terms P_{10}, P_{20} (expressed as a percentage) are given in Equation (38) as

h. Random Signal Processing, D. F. Mix, pp. 208–213, Prentice Hall, 1995.

$$\%P_{10} = P_{1FS} errP_1\%$$

$$\%P_{20} = P_{2FS} errP_2\% \ .$$

Thus, the assumptions that enabled Equation (38), i.e., the independence of parameters P_1, P_2 and moving the partial derivative operator inside the integral operation, are considered valid.

3.2 Application to System-Derived Parameters

The derived parameters of interest for INEEL testing are power, W; energy, E; capacity, Q; source impedance, R_s; efficiency, Eff; self-discharge, SD; and pulse power capability for both discharge and regen, R_D and P_R . All of these derived parameters have as independent variables some combination of voltage, V; current, I; time, t; or, possibly, other derived parameters. The relationships for the derived parameters (obtained as described in Section 1.4.2) are given by the following expressions:[i]

Power: $W = VI$ (45)

Energy: $E = \int VIdt$ (46)

Capacity: $Q = \int Idt$ (47)

Source impedance: $R_S = \dfrac{V(t_1) - V(t_2)}{I(t_1) - I(t_2)}$ (48)

Efficiency: $Eff = 100 \dfrac{E_{DISCHARGE}}{E_{CHARGE}} = 100 \dfrac{\int_{DISCHARGE} V_A I_A dt}{\int_{CHARGE} V_B I_B dt}$ (49)

Self-discharge:[j] $SD = \dfrac{(E_1 - E_A - E_B)}{7}$ (50)

Discharge pulse power capability: $P_D = \dfrac{V_{MIN}(OCV - V_{MIN})}{R_S}$ (51)

Regen pulse power capability: $P_R = \dfrac{V_{MAX}(V_{MAX} - OCV)}{R_S}$ (52)

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i. See the *PNGV Battery Testing Manual*, Revision 3, DOE/ID-10597 (published February 2001) for detailed descriptions of the calculations of R_s, Eff, SD, P_D, and P_R.

j. Note that the three subscripts *1*, *A,* and *B* here refer to the three parts of the self-discharge test. Specifically, part *1* is the uninterrupted discharge, and parts *A* and *B* are the parts of the interrupted discharge before and after the stand interval.

where

$V(t_1)$, $V(t_2)$, V_A, V_B	=	voltage at a specified time (t_1, t_2) or test condition (A, B)
$I(t_1)$, $I(t_2)$, I_A, I_B	=	current at a specified time (t_1, t_2) or test condition (A, B)
E_1, E_A, E_B	=	energy for specific test condition (1, A, B)
W	=	power (normally in kW)
E	=	energy (normally in W·h or kW·h)
Q	=	capacity (normally in A·h)
R_S	=	source resistance (normally in ohms, Ω)
Eff	=	round-trip efficiency (in %)
SD	=	self-discharge (normally in Wh/day, for a 7-day test; the constant divisor 7 would need to be altered for a test of a different duration.)
P_D and P_R	=	pulse power capability (normally in W)
V_{MIN}, V_{MAX}	=	specific voltages defined for the device under test using a particular test procedure
OCV	=	open circuit voltage measured or calculated at a specific point in a test procedure.

Normalizing a derived parameter's error relative to full-scale is not practical since full-scale is relevant only to measured parameters. Thus, the errors for derived parameters will be normalized to the actual value as a percentage of reading.

3.2.1 Power Uncertainty

The application of Equation (33) to the power relationship expressed in Equation (45) becomes

$$\%W = \frac{1}{V_{FS} I_{FS}} \left\{ \left(I V_{FS} \%err_V \right)^2 + \left(V I_{FS} \%err_I \right)^2 \right\}^{\frac{1}{2}} .$$

Expressed as a percentage of reading, the resulting uncertainty is given by

$$\%W = \left\{ \left(\frac{I_{FS}}{I} \%err_I \right)^2 + \left(\frac{V_{FS}}{V} \%err_V \right)^2 \right\}^{\frac{1}{2}} \qquad (53)$$

where

I and V	=	measured current and voltage

I_{FS} and V_{FS} = full-scale current and voltage

$\%err_I$ = uncertainty of the current (as a percentage of full scale)

$\%err_V$ = uncertainty of voltage (as a percentage of full scale).

3.2.2 Capacity Uncertainty

Equation (36) fits the derived parameter for capacity as expressed in Equation (54). The random error sources can be assumed to average to zero over time. However, the calibration error for a particular variable will not average out but will accumulate. Applying Equation (36) to capacity and choosing to define the uncertainty relative to reading, we obtain

$$\%Q = \frac{I_{FS}\%err_{ICAL}\int dt}{\int I(t)dt} \tag{54}$$

where

$\%err_{ICAL}$ = calibration error current (in % *FS*)

$\int I(t)dt$ = actual derived capacity *Q*.

Equation (54) can be further simplified to become Equation (55) by combining the two integral terms into average current:

$$\%Q = \frac{I_{FS}\%err_{ICAL}}{\bar{I}} \tag{55}$$

where

$\bar{I}$ = average current over the time interval

3.2.3 Energy Uncertainty

Rewriting Equation (38) specifically for the energy expression in Equation (46) yields

$$\%E = \frac{1}{E_{FS}}\sqrt{\left(\left\{\int Idt\right\}V_{FS}\%err_{VCAL}\right)^2 + \left(\left\{\int Vdt\right\}I_{FS}\%err_{ICAL}\right)^2} \ .$$

Expressed as a percentage of reading, the final result is given by

$$\%E = \frac{1}{E}\sqrt{\left(\left\{\int Idt\right\}V_{FS}\%err_{VCAL}\right)^2 + \left(\left\{\int Vdt\right\}I_{FS}\%err_{ICAL}\right)^2} \tag{56}$$

where

E = computed energy

I and *V*	=	measured current and voltage
I_{FS} and V_{FS}	=	full-scale current and voltage
$\%err_{VCAL}$	=	voltage calibration error (as % *FS*)
$\%err_{ICAL}$	=	current calibration error (as % *FS*).

3.2.4 Source Impedance Uncertainty

The source impedance relationship in Equation (48) has the general form of Equation (57):

$$F = \frac{P_1 - P_2}{P_3 - P_4} \ . \tag{57}$$

If all the parameters are statistically independent, then Equation (33) applies, and this relationship is

$$\%F = \frac{100}{F_{FS}}\left[\left\{\frac{\partial F}{\partial P_1}\Delta P_1\right\}^2 + \left\{\frac{\partial F}{\partial P_2}\Delta P_2\right\}^2 + \left\{\frac{\partial F}{\partial P_3}\Delta P_3\right\}^2 + \left\{\frac{\partial F}{\partial P_4}\Delta P_4\right\}^2\right]^{\frac{1}{2}} \ .$$

We must assume in implementing the above relationship that all the *cross derivatives* are zero. Consider what happens if we do not for the source impedance Equation 48.

Taking the partial derivatives of Equation (48) and including all the cross derivatives, we obtain

$$\frac{\partial \mathrm{R_S}}{\partial \mathrm{V}(\mathrm{t_1})} = \left(1 - \frac{\partial \mathrm{V}(\mathrm{t_2})}{\partial \mathrm{V}(\mathrm{t_1})}\right)(\mathrm{I}(\mathrm{t_1}) - \mathrm{I}(\mathrm{t_2}))^{-1} - (\mathrm{V}(\mathrm{t_1}) - \mathrm{V}(\mathrm{t_2}))(\mathrm{I}(\mathrm{t_1}) - \mathrm{I}(\mathrm{t_2}))^{-2}\left(\frac{\partial \mathrm{I}(\mathrm{t_1})}{\partial \mathrm{V}(\mathrm{t_1})} - \frac{\partial \mathrm{I}(\mathrm{t_2})}{\partial \mathrm{V}(\mathrm{t_1})}\right) \ . \tag{58}$$

We argue that the cross derivatives between parameters at different times are zero, and we define the term:

$$\frac{1}{\mathrm{R_S^*}} = \frac{\partial \mathrm{I}(\mathrm{t_1})}{\partial \mathrm{V}(\mathrm{t_1})} = \frac{\partial \mathrm{I}(\mathrm{t_2})}{\partial \mathrm{V}(\mathrm{t_2})} \tag{59}$$

$$R_S^* = \frac{\partial V(t_1)}{\partial I(t_1)} = \frac{\partial V(t_2)}{\partial I(t_2)} \ . \tag{60}$$

Combining Equations (58), (59), and (60),

$$\frac{\partial R_S}{\partial V(t_1)} = \left(1 - \frac{R_S}{R_S^*}\right)(I(t_1) - I(t_2))^{-1} \ .$$

In a similar way, the remaining partial derivatives can be obtained:

$$\frac{\partial \mathrm{R_S}}{\partial \mathrm{V(t_1)}} = -\left(1 - \frac{\mathrm{R_S}}{\mathrm{R_S^*}}\right)\left(\mathrm{I(t_1)} - \mathrm{I(t_2)}\right)^{-1}$$

$$\frac{\partial \mathrm{R_S}}{\partial \mathrm{I(t_1)}} = \left(\mathrm{R_S^*} - \mathrm{R_S}\right)\left(\mathrm{I(t_1)} - \mathrm{I(t_2)}\right)^{-1}$$

$$\frac{\partial \mathrm{R_S}}{\partial \mathrm{I(t_1)}} = -\left(\mathrm{R_S^*} - \mathrm{R_S}\right)\left(\mathrm{I(t_1)} - \mathrm{I(t_2)}\right)^{-1} .$$

If cross derivative terms are considered valid, then $R_S = R_S^*$, and the results are meaningless. We argue that the parameter variations that the partial derivative operator is sensing are due to the uncertainty of the measurement system, and such a variation in any one parameter will have no effect on any of the other parameters except the overall function. Thus, the existence of the term R_S^* can come only from real perturbations in the actual physical process, not from uncertainty injected into the data acquired by the measurement system. This proves that the cross derivatives do not exist, and the independence assumption is valid. Taking the partial derivatives of Equation (48) yields

$$\frac{\partial R_S}{\partial V(t_1)} = -\left[I(t_1) - I(t_2)\right]^{-1}, \frac{\partial R_S}{\partial V(t_2)} = \left[I(t_1) - I(t_2)\right]^{-1} \qquad (61)$$

$$\frac{\partial \mathrm{R_S}}{\partial \mathrm{I(t_1)}} = -\left[\mathrm{V(t_1)} - \mathrm{V(t_2)}\right]\left[\mathrm{I((t_1))} - \mathrm{I(t_2)}\right]^{-2}, \frac{\partial \mathrm{R_S}}{\partial \mathrm{I(t_2)}} = \left[\mathrm{V(t_1)} - \mathrm{V(t_2)}\right]\left[\mathrm{I(t_1)} - \mathrm{I(t_2)}\right]^{-2} . \qquad (62)$$

However, the voltages at times t_1 and t_2 in Equation (48) are not totally independent. The same is true of the currents. The steady-state errors consist of offset and sensitivity errors that are lumped and relative to full scale (V_{OS}, I_{OS}).[k] If we assume the worst case and consider that error to be all associated with sensitivity, then the differences between the voltage and current at times t_1 and t_2 will also generate errors and thus must also be considered as additional variables. For purposes of error analysis, the function of source impedance variables is given by

$$\mathrm{R_S} = \mathrm{R_S}\left[\mathrm{V(t_1)}, \mathrm{V(t_2)}, \mathrm{I(t_1)}, \mathrm{I(t_2)}, \Delta\mathrm{V(t_1, t_2)}, \Delta\mathrm{I(t_1, t_2)}\right] .$$

To obtain the additional partial derivatives, Equation (48) must be rewritten in the following forms:

$$R_S = \frac{\Delta V(t_1, t_2)}{I(t_1) - I(t_2)} \text{ and } R_S = \frac{V(t_1) - V(t_2)}{\Delta I(t_1, t_2)}$$

where $\Delta V(t_1, t_2) = V(t_1) - V(t_2)$, $\Delta I(t_1, t_2) = I(t_1) - I(t_2)$.

k. i.e., V_{OS} is the error voltage (in volts) at full scale that results from the combination of offset and sensitivity errors; and similarly for I_{OS}.

The additional partial derivatives for $\Delta V(t_1,t_2)$, $\Delta I(t_1,t_2)$ are given by

$$\frac{\partial R_S}{\partial \Delta V(t_1,t_2)} = [I(t_1) - I(t_2)]^{-1}, \quad \frac{\partial R_S}{\partial \Delta I(t_1,t_2)} = -[V(t_1) - V(t_2)][I(t_1) - I(t_2)]^{-2} \ . \tag{63}$$

The error associated with these additional variables is given by

$$\Delta V(t_1,t_2) = V_{OS} \frac{V(t_1) - V(t_2)}{V_{FS}}, \quad \Delta I(t_1,t_2) = I_{OS} \frac{I(t_1) - I(t_2)}{I_{FS}} \ . \tag{64}$$

The error associated with the individual voltage and current variables is given by

$$\Delta V(t_1) = \Delta V(t_2) = \sigma_V, \quad \Delta I(t_1) = \Delta I(t_2) = \sigma_I \ . \tag{65}$$

All of these errors are assumed statistical and independent, and substituting Equations (61), (62), (63), (64), and (65) into the Taylor Series statistical form (RSS) yields

$$\Delta R_S = \left\{ 2\left(\frac{\sigma_V}{I(t_1) - I(t_2)} \right)^2 + 2\left(\sigma_I \frac{[V(t_1) - V(t_2)]}{[I(t_1) - I(t_2)]^2} \right)^2 + \left(V_{OS} \frac{V(t_1) - V(t_2)}{V_{FS}[I(t_1) - I(t_2)]} \right)^2 + \left(I_{OS} \frac{V(t_1) - V(t_2)}{I_{FS}[I(t_1) - I(t_2)]} \right)^2 \right\}^{\frac{1}{2}} \ .$$

Dividing both sides of the above relation by R_S,

$$\frac{\Delta R_S}{R_S} = \left\{ 2\left(\frac{\sigma_V}{V(t_1) - V(t_2)} \right)^2 + 2\left(\frac{\sigma_I}{I(t_1) - I(t_2)} \right)^2 + \left(\frac{V_{OS}}{V_{FS}} \right)^2 + \left(\frac{I_{OS}}{I_{FS}} \right)^2 \right\}^{\frac{1}{2}} \ .$$

Converting to percentage of reading,

$$\%R_S = 100\left[2\left(\frac{\sigma_V}{V(t_1) - V(t_2)} \right)^2 + 2\left(\frac{\sigma_I}{I(t_1) - I(t_2)} \right)^2 + \left(\frac{V_{OS}}{V_{FS}} \right)^2 + \left(\frac{I_{OS}}{I_{FS}} \right)^2 \right]^{\frac{1}{2}} \ .$$

The final result is given by

$$\%R_S = \left[2\left(\frac{\%errV_{STD}}{V(t_1) - V(t_2)} V_{FS} \right)^2 + 2\left(\frac{\%errI_{STD}}{I(t_1) - I(t_2)} I_{FS} \right)^2 + (\%err_{VCAL})^2 + (\%err_{ICAL})^2 \right]^{\frac{1}{2}} \tag{66}$$

where

$\%errV_{STD}$	=	standard deviation for voltage uncertainty (as % *FS*)
$\%errI_{STD}$	=	standard deviation for current uncertainty (as % *FS*)
$V(t_1)$ and $V(t_2)$	=	measured voltages at times t_1 and t_2

$I(t_1)$ and $I(t_2)$ = measured currents at times t_1 and t_2

$\%err_{ICAL}$ = current calibration error (as % FS)

$\%err_{VCAL}$ = voltage calibration error (as % FS).

As with the uncertainty of energy, the uncertainty of source impedance given by Equation (66) is expressed as percentage of reading. Note that in this case it may be necessary to use $\%Ieq_{TOT}$ and $\%Veq_{TOT}$ as estimates of the current and voltage standard deviations.

3.2.5 Efficiency

Starting with Equation (33)[l] and the efficiency relationship in Equation (49), we obtain

$$\Delta Eff = \sqrt{\left(\frac{\partial Eff}{\partial V_A}\Delta V_A\right)^2 + \left(\frac{\partial Eff}{\partial V_B}\Delta V_B\right)^2 + \left(\frac{\partial Eff}{\partial I_A}\Delta I_A\right)^2 + \left(\frac{\partial Eff}{\partial I_B}\Delta I_B\right)^2} \ .$$

The partial derivatives are as follows:

$$\frac{\partial Eff}{\partial V_A} = 100\int I_A dt\left[\int V_B I_B dt\right]^{-1}, \quad \frac{\partial Eff}{\partial I_A} = 100\int V_A dt\left[\int V_B I_B dt\right]^{-1}$$

$$\frac{\partial Eff}{\partial V_B} = -100\left\{\int V_A I_A dt\left[\int V_B I_B dt\right]^{-2}\left[\int I_B dt\right]\right\}, \quad \frac{\partial Eff}{\partial I_B} = -100\left\{\int V_A I_A dt\left[\int V_B I_B dt\right]^{-2}\left[\int V_B dt\right]\right\} .$$

l. Note that Equation (33) applies, provided that the errors (including calibration errors) in parts A and B of the test are independent.

Substituting Equation (49) into each of the above relationships yields

$$\frac{\partial Eff}{\partial V_A} = Eff \frac{\int I_A dt}{\int V_A I_A dt}$$

$$\frac{\partial Eff}{\partial \mathrm{I_A}} = Eff \frac{\int \mathrm{V_A\,dt}}{\int V_A I_A dt}$$

$$\frac{\partial Eff}{\partial V_B} = -Eff \frac{\int I_B dt}{\int V_B I_B dt}$$

$$\frac{\partial Eff}{\partial I_B} = -Eff \frac{\int V_B dt}{\int V_B I_B dt}.$$

Since *Eff* is the result of time integrals, the standard deviation portion of the errors averages to zero, leaving only the calibration part of any errors, as follows:

$$\Delta V_A = \Delta V_B = \left(\%err_{VCAL} \times 10^{-2} \times V_{FS}\right)$$

$$\Delta I_A = \Delta I_B = \left(\%err_{ICAL} \times 10^{-2} \times I_{FS}\right)$$

where

$\%err_{VCAL}$	=	voltage calibration error (as % FS)
$\%err_{ICAL}$	=	current calibration error (as % FS)
I_{FS} and V_{FS}	=	full-scale current and voltage.

Combining these results, dividing by the calculated value of efficiency and converting to percentage of reading yields

$$\%Eff = \left\{ \left(V_{FS}\,\%err_{VCAL}\right)^2 \left(\left(\frac{\int I_A dt}{\int V_A I_A dt} \right)^2 + \left(\frac{\int I_B dt}{\int V_B I_B dt} \right)^2 \right) + \left(I_{FS}\,\%err_{ICAL}\right)^2 \left(\left(\frac{\int V_A dt}{\int V_A I_A dt} \right)^2 + \left(\frac{\int V_B dt}{\int V_B I_B dt} \right)^2 \right) \right\}^{\frac{1}{2}} . \quad (67)$$

Equation (67) can be simplified by assuming the following approximate relationships:

$$\frac{\int I_k dt}{\int V_k I_k dt} \approx \frac{\overline{I_k}}{\overline{\overline{P_k}}} = \frac{1}{\overline{\overline{\mathrm{V_k}}}}, \quad \frac{\int V_k dt}{\int V_k I_k dt} \approx \frac{\overline{V_k}}{\overline{\overline{P_k}}} = \frac{1}{\overline{\overline{I_k}}}$$

where the $\overline{V_k}$, $\overline{I_k}$, and $\overline{P_k}$ are averages over the time interval T_k. Equation (67) becomes the approximate result of Equation (68):

$$\Delta Eff \approx \left\{ \left(V_{FS}\,\%err_{VCAL}\right)^2 \left(\left(\frac{1}{\overline{V_A}}\right)^2 + \left(\frac{1}{\overline{V_B}}\right)^2 \right) + \left(I_{FS}\,\%err_{ICAL}\right)^2 \left(\left(\frac{1}{\overline{I_A}}\right)^2 + \left(\frac{1}{\overline{I_B}}\right)^2 \right) \right\}^{\frac{1}{2}} . \qquad (68)$$

Note that the goodness of this approximation is date-dependent. In the general case, Equation (67) should be used.

3.2.6 Self-Discharge

The expression for self-discharge given by Equation (50) can be rewritten in the form

$$SD = \frac{\left(E_1(\Delta t_1) - E_A(\Delta t_A) - E_B(\Delta t_B)\right)}{7}$$

where

$$E_1(\Delta t_1) = \int_{\Delta t_1} VIdt$$

$$E_A(\Delta t_A) = \int_{\Delta t_A} VIdt$$

$$E_B \Delta t_B = \int_{\Delta t_B} VIdt$$

and the integrals are performed over the three specific parts of the test defined in Section 3.2 (i.e., the interval Δt_1 is the duration of part 1 of the test, and correspondingly for parts A and B of the test).

Consider the general form of these energy terms:

$$E_i(\Delta t_i) = \int_{\Delta t_i} VIdt \ .$$

An error can be associated with each term and is given by

$$E_1\,(\Delta t_1) = E_i + err_i = \int_{\Delta t_i} (V + V_{ERR})(I + I_{ERR})dt \ . \qquad (69)$$

The expression for self-discharge can also be rewritten as a combination of these constituent errors for each term as

$$SD + \Delta SD = \frac{1}{7}\left[E_1 + err_1 - E_A - err_A - E_B - err_B\right] .$$

Examining this expression yields

$$\Delta SD = \frac{1}{7}\left[err_1 - err_A - err_B\right] . \quad (70)$$

From this point forward, there are two different approaches that can be taken. One approach assumes that error in voltage or current is due to a calibration offset. The other approach assumes that the error comes from linearity. In the earlier derivation of the uncertainty for measured parameters, the error associated with a parameter came from calibration and from random statistical noise. Because of the integrals, the noise term will be averaged to zero. However, the calibration error term will not. In the discussion for the derivation of the measured parameter relationships, the calibration error term was a combination of offset and linearity referenced to full scale. The two were lumped into a single error term because there were no available data that would allow for separation into offset and sensitivity errors. For some of the other derived parameters, there is no impact and hence no need to distinguish the nature of this error. However, for source impedance, because the offsets would subtract to zero it was necessary to select the conservative assumption that the calibration error was from sensitivity, not from offset. In the present case, it is not obvious which assumption is most conservative, and thus the derivation is presented for both cases. The user must therefore compute both uncertainties and select the most conservative result. (The general case, where errors result from both offset and linearity, is not treated here due to its complexity. In most cases, one of these effects will dominate the results.)

3.2.6.1 Case 1. Error Results Entirely from Calibration Offsets. Consider the power term in the integrand of Equation (69):

$$(V + V_{err})(I + I_{err}) = (V + V_{OS})(I + I_{OS}) = VI + V_{OS}I + I_{OS}V + I_{OS}V_{OS} . \quad (71)$$

These errors are considered to be only from calibration offsets, and the error terms V_{OS} and I_{OS} can be expressed as

$$V_{OS} = 0.01V_{FS}\%err_{VCAL} \quad (72)$$

$$I_{OS} = 0.01I_{FS}\%err_{ICAL} .$$

Substituting these expressions into the integrand, it becomes

$$VI + 0.01V_{FS}\%err_{VCAL}I + 0.01I_{FS}\%err_{ICAL}V + (0.01I_{FS}\%err_{ICAL})(0.01V_{FS}\%err_{VCAL})$$

where

$\%err_{VCAL}$	=	calibration voltage error (as % *FS*)
$\%err_{ICAL}$	=	calibration current error (as % *FS*)
V_{FS}	=	full-scale voltage used for voltage calibration
I_{FS}	=	full-scale current used for current calibration
V,I	=	measured voltage and current, respectively.

The last term in the integrand, $I_{OS}V_{OS}$, or $(0.01I_{FS}\%err_{ICAL})(0.01V_{FS}\%err_{VCAL})$, is neglected because it is the product of two very small quantities.

Substituting this reduced integrand into Equation (69) yields

$$E_i + err_i = \int_{\Delta t_i} (V + V_{OS})(I + I_{OS})dt \cong E_i + V_{OS} \int_{\Delta t_i} Idt + I_{OS} \int_{\Delta t_i} Vdt \ .$$

Thus, err_i can be expressed as

$$err_i = V_{OS} \int_{\Delta t_i} Idt + I_{OS} \int_{\Delta t_i} Vdt \ . \qquad (73)$$

Substituting Equation (73) into Equation (70) and simplifying, we obtain

$$\Delta SD = \frac{1}{7}\left[V_{OS}\left\{ \int_{\Delta t_1} Idt - \int_{\Delta t_A} Idt - \int_{\Delta t_B} Idt \right\} + I_{OS}\left\{ \int_{\Delta t_1} Vdt - \int_{\Delta t_A} Vdt - \int_{\Delta t_B} Vdt \right\}\right] .$$

In the above expression, the terms V_{os} and I_{os} are assumed to be statistically independent. Thus, changing to percentage of reading, we obtain

$$\%SD = \frac{100}{7SD}\left[(V_{OS})^2 \left\{ \int_{\Delta t_1} Idt - \int_{\Delta t_A} Idt - \int_{\Delta t_B} Idt \right\}^2 + (I_{OS})^2 \left\{ \int_{\Delta t_1} Vdt - \int_{\Delta t_A} Vdt - \int_{\Delta t_B} Vdt \right\}^2 \right]^{\frac{1}{2}} . \qquad (74)$$

Substituting the specific expressions for V_{os} and I_{os} into Equation (74) yields Equation (75) for this case:

$$\frac{1}{7SD}\left[(V_{FS}\%err_{VCAL})^2 \left\{ \int_{\Delta t_1} Idt - \int_{\Delta t_A} Idt - \int_{\Delta t_B} Idt \right\}^2 + (I_{FS}\%err_{ICAL})^2 \left\{ \int_{\Delta t_1} Vdt - \int_{\Delta t_A} Vdt - \int_{\Delta t_B} Vdt \right\}^2 \right]^{\frac{1}{2}} \qquad (75)$$

Noting that the first three integral terms in this result correspond to capacity values, this result can also be represented as Equation (76):

$$\%SD = \frac{1}{7SD}\left[(V_{FS}\%err_{VCAL})^2 (Q_1 - Q_A - Q_B)^2 + (\mathrm{I_{FS}}\%err_{ICAL})^2 \left\{ \int_{\Delta t_1} Vdt - \int_{\Delta t_A} Vdt - \int_{\Delta t_B} Vdt \right\}^2 \right]^{\frac{1}{2}} \qquad (76)$$

where Q_1, Q_A, and Q_B are the capacities measured[k] during the three specific parts (1, A, and B) of the self-discharge test, as defined in Section 3.2.

This expression applies provided that the same current channel is used for the measurement of both energy and capacity, which is normally the case.

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k. Using the same time units as self-discharge, i.e., if energy loss is measured in W·h, then these capacities are measured in A·h.

3.2.6.2 Case 2. Error Results Entirely from Calibration Linearities.

Again, consider the integrand of Equation (69):

$$(V + V_{ERR})(I + I_{ERR}) = (V + V_{LIN})(I + I_{LIN}) = VI + V_{LIN}I + I_{LIN}V + I_{LIN}V_{LIN} \;.$$

This error is now considered to be only from linearity, and the error terms, V_{LIN} and I_{LIN}, can be expressed as

$$V_{LIN} = 0.01\%err_{VCAL}V$$

$$I_{LIN} = 0.01\%err_{ICAL}I \;.$$

Substituting these expressions into the integrand, it becomes

$$VI + 0.01\%err_{VCAL}VI + 0.01\%err_{ICAL}IV + (0.01\%err_{ICAL})(0.01\%err_{VCAL})VI$$

where

$\%err_{VCAL}$	=	calibration sensitivity error for voltage *as a percentage of reading*
$\%err_{ICAL}$	=	calibration sensitivity error for current *as a percentage of reading*
V and *I*	=	measured voltage and current, respectively.

As in Case 1, the last term is neglected because it is the product of very small numbers. Thus, Equation (69) can be expressed as

$$E_i + err_i = \int_{\Delta t_i} (VI + 0.01\%err_{VCAL}VI + 0.01\%err_{ICAL}IV)dt$$

$$err_i = 0.01\%err_{VCAL}\int_{\Delta t_i}(VI)dt + 0.01\%err_{ICAL}\int_{\Delta t_i}(IV)dt$$

$$err_i = (0.01\%err_{VCAL} + 0.01\%err_{ICAL})\int_{\Delta t_i}(IV)dt$$

$$err_i = (0.01\%err_{VCAL} + 0.01\%err_{ICAL})E_i \;.$$

Because the voltage and current errors are assumed to be statistically independent, the above expression becomes

$$err_i = \left((0.01\%err_{VCAL})^2 + (0.01\%err_{ICAL})^2\right)^{\frac{1}{2}} E_i \;. \qquad (77)$$

Substituting Equation (77) into the general expression for the self-discharge error in Equation (70), we obtain

$$\Delta SD = \frac{1}{7}\left[E_1 - E_A - E_B\right]\left(\left(0.01\%err_{VCAL}\right)^2 + \left(0.01\%err_{ICAL}\right)^2\right)^{\frac{1}{2}}$$

$$\%SD = 100\frac{\Delta SD}{SD} = \left(\left(\%err_{VCAL}\right)^2 + \left(\%err_{ICAL}\right)^2\right)^{\frac{1}{2}} \ . \qquad (78)$$

The final result in Equation (78) expresses the error in self-discharge (as a percentage of reading) caused from linearity error (also as a percentage of reading).

3.2.7 Pulse Power Capability

Pulse power capability is a derived parameter calculated from a combination of directly measured parameters (voltage and current) and other derived parameters (source resistance, and in some cases an interpolated voltage value). Pulse power capability at a given depth-of-discharge is calculated separately for discharge and regen conditions. Both cases are treated in this section. The expressions used for the calculations are shown in Equations (51) and (52). They are similar in form, both involving the product of a limiting voltage and a voltage difference (proportion to the maximum allowable current at the present DOD) divided by the applicable source resistance.

The limiting voltage is a constant specified by the battery manufacturer. The voltage difference corresponds to the maximum allowable voltage change from the present open-circuit voltage (OCV) to the limiting voltage. For discharge pulse power capability, the applicable OCV is measured just prior to the start of the test current pulse and is thus a directly measured value. For regen pulse power capability, the battery is not at voltage equilibrium at the start of the test current pulse; consequently, the OCV is interpolated from the previous and succeeding OCV values measured during the test, based on the relative charge fraction removed before and after the test current pulse.[1]

3.2.7.1 Discharge Pulse Power Capability. For discharge conditions, pulse power capability is calculated using the following equation:

$$Disharge\ Pulse\ Power\ Capability = V_{MIN} \cdot \left(OCV_{DIS} - V_{MIN}\right) \div R_{DISCHARGE}$$

Restating the Discharge Power expression with the variables in abbreviated form,

$$P_D = V_{MIN} \cdot \left(V_0 - V_{MIN}\right) \div R_S \qquad (79)$$

where

$$R_S = \frac{\left(V_0 - V_1\right)}{\left(I_0 - I_1\right)} \ . \qquad (80)$$

In equation (79), P_D is a function of four measured variables. The uncertainty of P_D is affected by these variables along with the differences of pairs of these variables:

$$P_D\left\{V_0, V_1, I_0, I_1, \left(V_0 - V_1\right), \left(I_0 - I_1\right)\right\}$$

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1. See page 28 of the *PNGV Battery Test Manual* (DOE/ID-10597, Rev. 3, Feb. 2001) for a detailed description of this test and associated calculations.

where

V_0 = measured voltage at time t_0 just prior to the discharge current pulse

V_1 = measured voltage at specified time t_1 during the discharge current pulse

I_0 = measured current at time t_0 just prior to the discharge current pulse (normally zero)

I_1 = measured current at specified time t_1 during discharge current pulse

$(V_0 - V_1)$ = change in measured voltage from time t_0 to time t_1

$(I_0 - I_1)$ = change in measured current from time t_0 to time t_1

V_{MIN} = manufacturer-defined constant that represents the minimum voltage allowed under pulse discharge conditions.

As with the derivation for source impedance in Section 3.2.4, we make the conservative assumption that the calibration error is due to sensitivity (i.e., linearity), and thus both the voltage difference and the current difference become parameters that introduce uncertainty. (For the offset portion of such error, the differences would not contribute error to the power capability determination.) Substituting Equation (80) into Equation (79) and obtaining the partial derivatives,

$$\frac{\partial P_D}{\partial V_0} = \frac{V_{MIN}}{R_S}\frac{(V_{MIN} - V_1)}{(V_0 - V_1)}$$

$$\frac{\partial P_D}{\partial V_1} = \frac{V_{MIN}}{R_S}\frac{(V_0 - V_{MIN})}{(V_0 - V_1)}$$

$$\frac{\partial P_D}{\partial I_0} = V_{MIN}\frac{(V_0 - V_{MIN})}{(V_0 - V_1)}$$

$$\frac{\partial P_D}{\partial I_1} = V_{MIN}\frac{(V_0 - V_{MIN})}{(V_0 - V_1)}$$

$$\frac{\partial P_D}{\partial (V_0 - V_1)} = \frac{-V_{MIN}}{R_S}\frac{(V_0 - V_{MIN})}{(V_0 - V_1)}$$

$$\frac{\partial P_D}{\partial (I_0 - I_1)} = V_{MIN}\frac{(V_0 - V_{MIN})}{(V_0 - V_1)} \; .$$

Each of these partial derivatives is used in the Taylor Series formulation of uncertainty, limiting terms to first derivatives only and assuming statistical independence:

$$\Delta P_D = \left\{ \begin{array}{l} \left(\frac{\partial P_D}{\partial V_0}\right)^2 \left(\Delta V_{OS}^2 V_{FS}^2 + \sigma_V^2\right) + \left(\frac{\partial P_D}{\partial V_1}\right)^2 \sigma_V^2 + \left(\frac{\partial P_D}{\partial \mathrm{I}_0}\right)^2 \sigma_I^2 + \left(\frac{\partial P_D}{\partial I_1}\right)^2 \sigma_I^2 + \\ \left(\frac{\partial P_D}{\partial (V_0 - V_1)}\right)^2 \Delta V_{OS}^2 (V_0 - V_1)^2 + \left(\frac{\partial P_D}{\partial (I_0 - I_1)}\right)^2 \Delta I_{OS}^2 (I_0 - I_1)^2 \end{array} \right\}^{\frac{1}{2}} . \quad (81)$$

Note that because the variable V_0 appears in the expression by itself as well as in a difference term, it contributes calibration error, conservatively expressed as an offset given by $\Delta V_{OS} V_{FS}$, along with the standard deviation error of σ_V . Substituting the partial derivatives into Equation (81) and simplifying,

$$\Delta P_D = \left\{ \begin{array}{l} \left(\frac{\sigma_V V_{MIN}}{R_S (V_0 - V_1)}\right)^2 \left[(V_{MIN} - V_1)^2 + (V_0 - V_{MIN})^2\right] + 2\sigma_I^2 V_{MIN}^2 \left(\frac{V_0 - V_{MIN}}{V_0 - V_1}\right)^2 + \\ \frac{\Delta V_{OS}^2 V_{FS}^2 V_{MIN}^2}{R_S^2} \left(\frac{V_{MIN} - V_1}{V_0 - V_1}\right)^2 + \frac{V_{MIN}^2}{R_S^2} (V_0 - V_{MIN})^2 \left(\Delta I_{OS}^2 + \Delta V_{OS}^2\right) \end{array} \right\}^{\frac{1}{2}} . \quad (82)$$

Rearranging terms in Equation (82) and dividing by P_D gives the result as a relative value, i.e., a fraction of reading:

$$\frac{\Delta P_D}{P_D} = \sqrt{\Delta V_{OS}^2 + \Delta I_{OS}^2 + \frac{\sigma_V^2}{(V_0 - V_1)^2} + \frac{2\sigma_I^2}{(I_0 - I_1)^2} + \frac{\left(\sigma_V^2 + \Delta V_{OS}^2 V_{FS}^2\right)(V_{MIN} - V_1)^2}{(V_0 - V_1)^2 (V_0 - V_{MIN})^2}} . \quad (83)$$

Note that ΔV_{OS} and ΔI_{OS} in this derivation are the fractional error voltage and current due to calibration errors, i.e., $\Delta V_{OS} = 0.01\% \, errV_{CAL}$, and $\Delta I_{OS} = 0.01\% \, errI_{CAL}$.

Converting Equation (83) to percentage of reading and substituting the percentage error terms used in previous derivations yields the final result of Equation (84):

$$\%P_D = \left\{ \begin{array}{l} (\%err_{VCAL})^2 + (\%err_{ICAL})^2 + \frac{\%errV_{STD}^2 V_{FS}^2}{(V_0 - V_1)^2} + 2\frac{\%errI_{STD}^2 \mathrm{I}_{FS}^2}{(I_0 - I_1)^2} \\ + \frac{\left(\%errV_{STD}^2 V_{FS}^2 + \%err_{VCAL}^2 V_{FS}^2\right)(V_{MIN} - V_1)^2}{(V_0 - V_1)^2 (V_0 - V_{MIN})^2} \end{array} \right\}^{\frac{1}{2}} \quad (84)$$

where

$\%errV_{STD}$	=	standard deviation for voltage uncertainty (as %*FS*)
$\%errI_{STD}$	=	standard deviation for current uncertainty (as %*FS*)
V_{FS}	=	full-scale voltage

I_{FS} = full-scale current

$\%err_{ICAL}$ = current calibration error (as $\%FS$)

$\%err_{VCAL}$ = voltage calibration error (as $\%FS$).

3.2.7.2 Regen Pulse Power Capability. For regen conditions, pulse power capability is calculated using the following expression:

$$Regen\ Pulse\ Power\ Capability = V_{MAX} \cdot (V_{MAX} - OCV_{REGEN}) \div R_{REGEN} \ .$$

Restating the regen power capability expression with the variables in abbreviated form,

$$P_R = V_{MAX} \cdot (V_{MAX} - V_R) \div R_S \qquad (85)$$

where

$$R_S = \frac{(V_2 - V_3)}{(I_2 - I_3)} \qquad (86)$$

$$V_R = V_4 - (V_4 - V_5)\frac{Q_A}{Q_A + Q_B} \qquad (87)$$

$$Q_A = \int_{T_A} I dt\ ,\ Q_B = \int_{T_B} I dt\ \ . \qquad (88)$$

The basic form of Equation (85) is the same as Equation (79). However, V_R is not a measured variable; rather, it is a derived variable used only to support this calculation. In Equation (85), P_R is a function of the following measured variables (and/or differences of measured variables) that determine its uncertainty:

$$P_D \{I, V_2, V_3, V_4, V_5, I_2, I_3, (V_2 - V_3), (I_2 - I_3), (V_4 - V_5)\}$$

where

I = measured current to be integrated during time intervals T_A and T_B during the test

T_A = time interval from the start of the previous discharge pulse to just prior to the regen current pulse

T_B = time interval from just prior to the regen current pulse to the start of the following discharge pulse

V_2 = measured voltage at time t_2 just prior to the regen current pulse

V_3 = measured voltage at a specified time t_3 during the regen current pulse

V_4	=	open-circuit voltage measured at the start of the previous discharge pulse
V_5	=	open-circuit voltage measured at the start of the following discharge pulse
I_2	=	measured current at time t_2 just prior to the regen current pulse
I_3	=	measured current at a specified time t_3 during the regen current pulse
$(V_2 - V_3)$	=	change in measured voltage from time t_2 to time t_3
$(I_2 - I_3)$	=	change in measured current from time t_2 to time t_3
$(V_4 - V_5)$	=	change in open-circuit voltage between start of previous and next discharge pulses
Q_A	=	net discharge during previous discharge pulse
Q_B	=	net discharge during regen pulse and following C/1 discharge segment (Q_A and Q_B normally total 10% DOD measured)
V_{MAX}	=	manufacturer-defined constant that represents the maximum voltage allowed under pulse regen conditions.

Substituting Equations (86) and (87) into Equation (85) and taking the appropriate partial derivatives of the variables that cause uncertainty, and using the assumption that

$$\frac{\partial Q_A}{\partial I} = T_A \text{ and } \frac{\partial Q_B}{\partial I} = T_B$$

yields the following:

$$\frac{\partial P_R}{\partial I} = \frac{V_{MAX}(V_4 - V_5)}{R_S} \frac{(T_A Q_B - T_B Q_A)}{(Q_A + Q_B)^2} .$$

Note that this partial derivative assumes that errors in I will apply equally during the time intervals T_A and T_B, which is not guaranteed to be true under all conditions. If the partial differentiation is done with respect to each time interval separately, the resulting partial derivatives are

$$\frac{\partial P_R}{\partial I_{T_A}} = \frac{V_{MAX}(V_4 - V_5)}{R_S} \frac{T_A Q_B}{(Q_A + Q_B)^2}, \quad \frac{\partial P_R}{\partial I_{T_B}} = \frac{V_{MAX}(V_4 - V_5)}{R_S} \frac{T_B Q_A}{(Q_A + Q_B)^2} . \tag{89}$$

Because the value of Q_B can be negative, the choice between these two forms of the partial derivative may be data-dependent. However, it can be shown that this second form is the conservative version over the current ranges most commonly used for the HPPC test, and thus it is used here.

$$\frac{\partial P_R}{\partial V_2} = -\frac{P_R}{(V_2 - V_3)}, \quad \frac{\partial P_R}{\partial V_3} = \frac{P_R}{(V_2 - V_3)} \tag{90}$$

$$\frac{\partial P_R}{\partial V_4} = -\frac{V_{MAX}}{R_S}\left(\frac{Q_B}{Q_A + Q_B}\right), \quad \frac{\partial P_R}{\partial V_5} = -\frac{V_{MAX}}{R_S}\left(\frac{Q_A}{Q_A + Q_B}\right) \tag{91}$$

$$\frac{\partial P_R}{\partial I_2} = \frac{P_R}{(I_2 - I_3)}, \quad \frac{\partial P_R}{\partial I_3} = -\frac{P_R}{(I_2 - I_3)} \tag{92}$$

$$\frac{\partial P_R}{\partial (V_2 - V_3)} = -\frac{P_R}{(V_2 - V_3)} \tag{93}$$

$$\frac{\partial P_R}{\partial (I_2 - I_3)} = \frac{P_R}{(I_2 - I_3)} \tag{94}$$

$$\frac{\partial P_R}{\partial (V_4 - V_5)} = \frac{V_{MAX}}{R_S}\left(\frac{Q_A}{Q_A + Q_B}\right). \tag{95}$$

The Taylor Series form of uncertainty for these partial derivatives is given by

$$\Delta P_R = \tag{96}$$

$$\left\{\begin{array}{c} \left(\frac{\partial P_R}{\partial I_{T_A}}\Delta I_{OS} I_{FS}\right)^2 + \left(\frac{\partial P_R}{\partial I_{T_A}}\Delta I_{OS} I_{FS}\right)^2 + \left(\frac{\partial P_R}{\partial V_2}\sigma_V\right)^2 + \left(\frac{\partial P_R}{\partial V_3}\sigma_V\right)^2 + \left(\frac{\partial P_R}{\partial (V_2 - V_3)}\Delta V_{OS}(V_2 - V_3)\right)^2 \\ + \left(\frac{\partial P_R}{\partial V_4}\sigma_V\right)^2 + \left(\frac{\partial P_R}{\partial V_4}\Delta V_{OS} V_{FS}\right)^2 + \left(\frac{\partial P_R}{\partial V_5}\sigma_V\right)^2 + \left(\frac{\partial P_R}{\partial (V_4 - V_5)}\Delta V_{OS}(V_4 - V_5)\right)^2 \\ + \left(\frac{\partial P_R}{\partial I_2}\sigma_I\right)^2 + \left(\frac{\partial P_R}{\partial I_3}\sigma_I\right)^2 + \left(\frac{\partial P_R}{\partial (I_2 - I_3)}\Delta I_{OS}(I_2 - I_3)\right)^2 \end{array}\right\}^{\frac{1}{2}}.$$

The general principle underlying this form of the Taylor series expansion is that individual parameters are affected by the complete error contribution, while integrals of parameters are affected only by the calibration error contribution. If the parameter is a difference of itself at two separate times, then the calibration error contribution is assumed to be entirely from linearity. If the statistics of the calibration errors are such that either offset or linearity errors are dominant, it may be possible to neglect some of these terms in practice.

Substituting the partial derivatives of Equations (89) through (95) into each term of Equation (96), regrouping by error type, simplifying the resulting expressions, and dividing by P_R to convert the result to fraction of reading gives Equation (97):

$$
\frac{\Delta P_R}{P_R} = \left\{ \begin{array}{c} \sigma_V^2 \left[\frac{2}{(V_2 - V_3)^2} + \frac{1}{(V_{MAX} - V_R)^2} \frac{Q_A^2 + Q_B^2}{(Q_A + Q_B)^2} \right] \\ + \Delta V_{OS}^2 \left[1 + \frac{(V_4 - V_5)^2 Q_A^2 + V_{FS}^2 Q_B^2}{(V_{MAX} - V_R)^2 (Q_A + Q_B)^2} \right] \\ + \frac{2\sigma_I^2}{(I_2 - I_3)^2} + \Delta I_{OS}^2 \left[1 + I_{FS}^2 \frac{(V_4 - V_5)^2 \left(T_A^2 Q_B^2 + T_B^2 Q_A^2 \right)}{(V_{MAX} - V_R)^2 (Q_A + Q_B)^4} \right] \end{array} \right\}^{\frac{1}{2}} \tag{97}
$$

where

σ_V = standard deviation for measured voltage (V)

σ_I = standard deviation for measured current (A)

ΔI_{OS} = combined current calibration error for measured current (unitless fraction)

ΔV_{OS} = combined voltage calibration error for measured voltage (unitless fraction)

I_{FS} = full-scale value for measured current (A)

V_{FS} = full-scale value for measured voltage (V).

In defense of the reasonableness of this very complicated expression, note that the first term associated with each error type is identical (allowing for notation differences in ΔV_{OS} and ΔI_{OS}) to that derived for source resistance in Section 3.2.4. The remaining terms all have to do with errors introduced by the calculation of the open-circuit voltage V_R. It is possible to derive the uncertainty of V_R separately using Equation (87) (because all of its associated variables are independent of others used in calculating P_R) and combine this result using a simplified version of the Taylor series expression in Equation (96). This gives the same result as Equation (97) but is not shown here due to its length.

The uncertainty parameters defined above can be expressed in terms of the general form of uncertainty parameters defined earlier for voltage and current:

$$
\sigma_V = \%errV_{STD} \frac{V_{FS}}{100}, \quad \sigma_I = \%errI_{STD} \frac{I_{FS}}{100}
$$

$$
\Delta I_{OS} = \frac{\%err_{ICAL}}{100}, \quad \Delta V_{OS} = \frac{\%err_{VCAL}}{100} .
$$

If these are substituted in Equation (97), the resulting Equation (98) gives the uncertainty of regen power capability as a percentage of reading:

$$\%P_R = \left\{ \begin{aligned} & (\%errV_{STD})^2 V_{FS}^2 \left[\frac{2}{(V_2 - V_3)^2} + \frac{Q_A^2 + Q_B^2}{(V_{MAX} - OCV_{REGEN})^2 (Q_A + Q_B)^2} \right] \\ & + (\%err_{VCAL})^2 \left[1 + \frac{(V_4 - V_5)^2 Q_A^2 + V_{FS}^2 Q_B^2}{(V_{MAX} - OCV_{REGEN})^2 (Q_A + Q_B)^2} \right] \\ & + \frac{2(\%errI_{STD})^2 I_{FS}^2}{(I_2 - I_3)^2} + (\%err_{ICAL})^2 \left[1 + I_{FS}^2 \frac{(V_4 - V_5)^2 (T_A^2 Q_B^2 + T_B^2 Q_A^2)}{(V_{MAX} - OCV_{REGEN})^2 (Q_A + Q_B)^4} \right] \end{aligned} \right\} . \tag{98}$$

Again, note that the first term associated with each error type is identical (except for minor notational changes) to the final result for Source Resistance uncertainty in Equation (66). We originally suspected that many or all of the other terms associated with errors in OCV_{REGEN} would be negligible, but preliminary calculations using sample data indicate that this is not the case. Consequently, the entire expression has been retained for completeness.

Application of Equation (98) must account for the fact that the time intervals T_A and T_B in Equation (98) are the times during which charge is integrated for the calculation of OCV_{REGEN}. For some types of battery testers, charge is not accumulated during rest periods, and thus current offsets during such intervals would not affect the results. In such cases, the relevant times are not exactly as defined in Equation (88); rather, they are only the portions of these intervals when charge is actually being accumulated.

3.2.8 Available Energy

There is one additional derived parameter of interest to the PNGV program that is not defined in Section 1.4.2, nor is there an expression provided for it in Section 3.2. This parameter is called *available energy*, and it has not been previously discussed because there is no explicit expression that defines it in terms of measured variables. It is defined implicitly by the method used for calculating it, which is described in Appendix E of the *PNGV Battery Test Manual*, Revision 3, DOD/ID-10597. This calculation approach means that determining uncertainty for available energy is a more complex process than for other derived parameters. It is not clear that it is even proper to refer to “measurement” uncertainty for this parameter, because no directly measured parameters are used in the calculation; instead, available energy is calculated from a series of values for other derived parameters, which in turn are calculated from two different tests.

A simplified description of available energy is as follows: it represents the energy available for discharge at a constant rate between two depth-of-discharge (DOD) values.[m] The two DOD values are (1) the minimum DOD value at which the battery’s calculated regen pulse power capability equals the PNGV regen pulse power goal; and (2) the maximum DOD value at which the battery’s calculated discharge pulse power capability equals the PNGV discharge pulse power goal. Discharge and regen pulse power capabilities are calculated from a Hybrid Pulse Power Characterization (HPPC) test, which provides data at nine specific DOD values for each. Discharge energy is calculated from data acquired during either a constant-current or a constant-power discharge test (abbreviated as constant-current/power

m. Depth-of-discharge (DOD) is the fraction of rated battery capacity removed from a fully charged battery under some prescribed test conditions. DOD = (charge removed) ÷ (rated capacity) test here). The results of these two tests are related by equating the corresponding DOD values in the two tests.

The meaningfulness of available energy depends on these assumptions: (a) a battery can be modeled simplistically but usefully as an ideal voltage source in series with an internal source resistance, (b) both the source voltage and the source resistance are continuous and “well behaved” functions of depth-of-discharge, and (c) a given DOD value represents the same battery “state” for both HPPC and constant-current/power tests. The source voltage and source resistance are functions of the specific battery under test and are thus not defined by exact expressions; instead, they are determined at various DOD values from test data. As a direct result of these assumptions and the expressions used for calculating them, discharge energy and pulse power capability are also continuous and “well behaved” functions of DOD. Thus, they can be related to each other through depth-of-discharge as an intermediate variable.

Note that discharge and regen pulse power capability are “known” only at the nine DOD values where calculations can be made for each, but determination of available energy is made using two specific pulse power capability values (1 discharge and 1 regen), which may not correspond to any of the “measured” points. For practical purposes, this requires fitting a curve through the nine calculated values and assuming that it represents the underlying function.[n] This is also true for the relationship between discharge energy and DOD, although many more data points are typically available in this case.

As a final result, available energy is wanted as a function of pulse power capability. This is accomplished in stepwise fashion by (1) determining the DOD values corresponding to the discharge and regen power capability goal values, (2) equating these HPPC DOD values to the corresponding constant current/power DOD values, and (3) determining the discharge energy values corresponding to these same DOD values. The difference between the two resulting discharge energy values is defined as *available energy*. This process is illustrated graphically in Figure 9, where the regen and discharge power goals are designated P_R and P_D; the corresponding DOD values are Q_R and Q_D; and the resulting energy values are E_R and E_D. Available energy is then the difference between E_D and E_R.

Three types of measurement error are inherent in this process: (1) uncertainty in the current and voltage measurements used to calculate pulse power capability, (2) uncertainty in the current measurement used to calculate DOD during the HPPC test and the constant current/power test, and (3) uncertainty in the current and voltage measurements used to calculate energy during the constant current/power test.[o]

Errors in determining DOD (i.e., type 2 above) are neglected here, because the available energy calculation process *defines* the measured DOD values in the two tests as equivalent.[p] Thus, the DOD values themselves do not contribute to uncertainty in the result; only the correspondence between the two DOD scales is used.

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n. Note that the PNGV goals are different for discharge and regen pulse power capability. The graphical approach to determining available energy described in the *PGNV* manual typically scales the regen power capability curve by the ratio of the two goals (so that both power curves are expressed in terms of equivalent discharge power). However, the uncertainties of discharge and regen power capabilities are different functions of both power and DOD. The description given here avoids this complexity by using the (different) goal power values directly.

o. DOD and energy are usually the results of numerical integration (of current or the product of current and voltage) within a battery test station and thus are calculated parameters.

p. This assumed equivalence of the two DOD scales is not electrochemically exact in any case. Current calibration errors are the only measurement errors that would affect this correspondence, and differences due to these are small because the two tests are run consecutively using the same current measurement channel.

Determining the uncertainty of available energy requires identifying the contributing error terms, accounting for any dependencies between them, and combining independent error terms in RSS fashion. There are four error sources to be considered: (1) $\Delta E_{D,P}$, the error in E_D due to uncertainty in the measurements used to calculate P_D, (2) $\Delta E_{R,P}$, the error in E_R due to uncertainty in the measurements used to calculate P_R, (3) the error in E_D due to uncertainty in the measurements used to calculate E_D, and (4) the error in E_R due to uncertainty in the measurements used to calculate E_R.

Error sources (3) and (4) can be combined, because Equation (56) shows that they both depend only on the voltage and current calibration errors over the respective time intervals where E_D and E_R are calculated. Since the time interval for (4) is a subset of the time interval for (3), the resulting combined error of these sources, designated ΔE_{D-R}, is found by using Equation (56) calculated over the portion of the constant current/power test between Q_R and Q_D.

$$\Delta E_{D-R} = \frac{1}{100}\sqrt{\left(\left\{\int Idt\right\}V_{FS}\%err_{VCAL}\right)^2 + \left(\left\{\int Vdt\right\}I_{FS}\%err_{ICAL}\right)^2} \tag{99}$$

where the integrals of I and V are computed between Q_R and Q_D and other terms are as in Equation (56).

All of ΔE_{D-R}, $\Delta E_{D,P}$, and $\Delta E_{R,P}$ are independent with respect to random measurement errors. They share some potential dependencies because the same voltage and current calibration errors apply to all of them. However, ΔE_{D-R} is affected primarily by calibration offset errors, while $\Delta E_{D,P}$ and $\Delta E_{R,P}$ are affected primarily by calibration linearity errors. Further, the shared effects of linearity errors on $\Delta E_{D,P}$ and $\Delta E_{R,P}$ can be neglected, provided that they are not the dominant error sources. Thus, these terms will be treated as independent for purposes of this study.[q]

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q. In some cases, these effects can be expressed more precisely. For example, current calibration linearity errors have no effect on a constant-current test, while neither current nor voltage calibration offset errors have any effect on discharge power capability results. In general, there must be assumed to be at least some weak dependencies among these error sources that are being neglected here, and thus an RSS calculation is not guaranteed to be conservative. However, in prior sections of this report calibration errors are generally assumed to be either all offset or all linearity (whichever is worse); either assumption in this case would tend to reduce the effects of these dependencies.

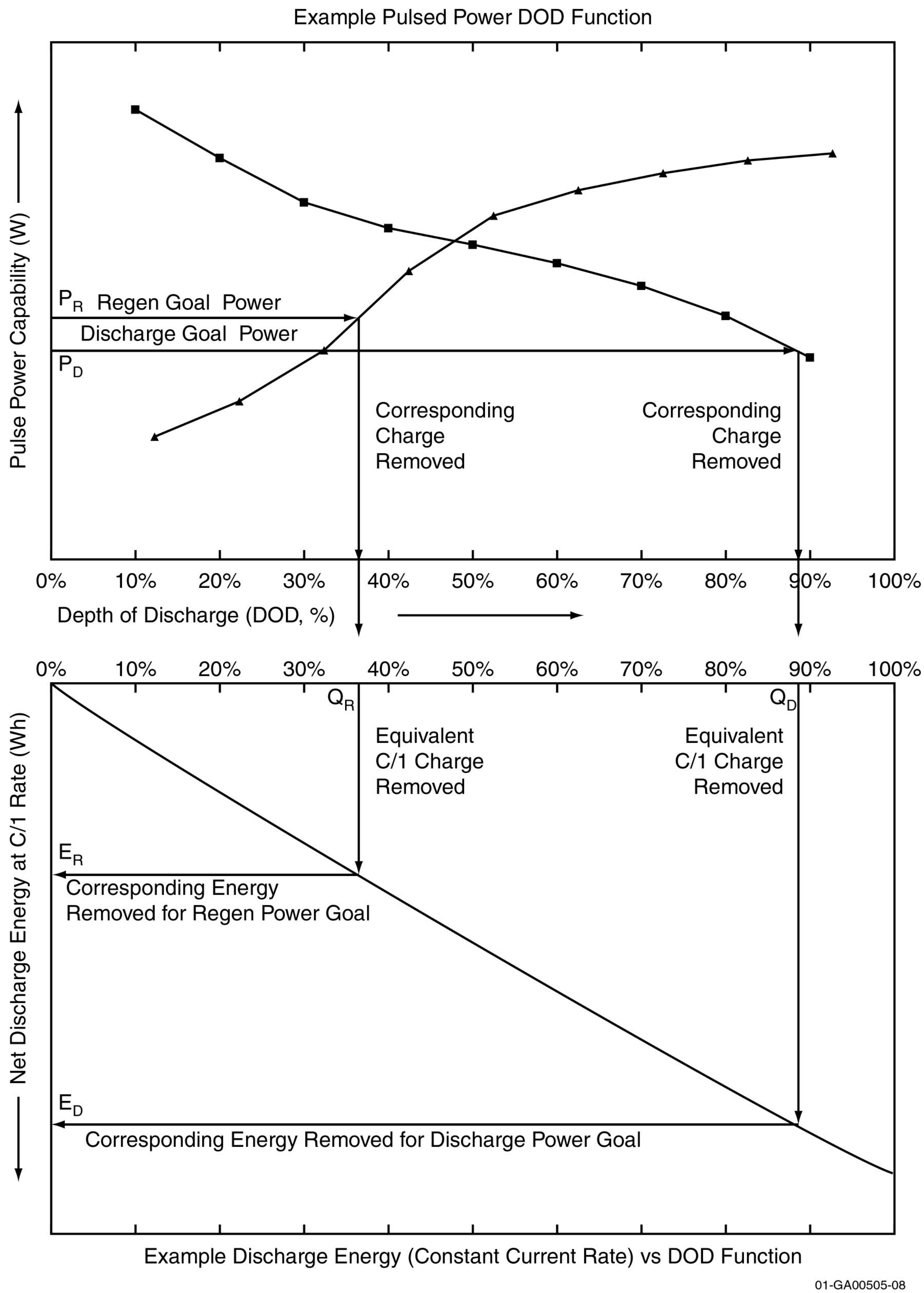

Figure 9. Available energy calculation process.

Based on this assumed independence, the error in available energy (in units of energy) can be expressed as Equation (100):

$$\Delta AE = \sqrt{(\Delta E_{D-R})^2 + (\Delta E_{D,P})^2 + (\Delta E_{R,P})^2} \quad . \tag{100}$$

Developing expressions for the other error terms, $\Delta E_{D,P}$ and $\Delta E_{R,P}$, is a somewhat more complex process. It requires visualizing the energy relationship in Figure 9 as a function of DOD (charge removed), which in turn is visualized as a function of either discharge or regen power capability (i.e., the inverse relationship of the upper curves in Figure 9.) Generically, this can be expressed in terms of three functions (*f*, *g,* and *h*), which are assumed to represent the underlying behavior in the three curves shown in Figure 9:

$$E = f(Q)$$
$$Q_{discharge} = g(P_{discharge}), \quad Q_{regen} = h(P_{regen})$$
$$E_D = f(g(P_D)), \quad E_R = f(h(P_R)).$$

Given these functional relationships (whose exact expressions are unknown, although analytical expressions can be derived for a given data set by fitting curves to the known points), it is now possible to estimate $\Delta E_{D,P}$ and $\Delta E_{R,P}$.

Three other functional relationships need to be defined: the uncertainties in energy, discharge power capability, and regen power capability as a function of DOD. Measurement uncertainties for these three derived parameters are defined by Equations (56), (84), and (98) respectively, but these are in terms of the underlying measurements of current and voltage. Neither the power capabilities of interest (P_D and P_R) nor the energy values of interest (E_D or E_R) necessarily correspond to any of the measured points, so the uncertainties at these points must be determined by fitting curves through the uncertainty values calculated at the measured points. Examples of the resulting uncertainty relationships are shown in Figures 10 and 11 for the same sample data set used in Figure 9.

The sample energy uncertainty function in Figure 10 appears to have the same shape as the energy curve itself in Figure 9. This is because the expression derived for energy uncertainty in Equation (56) is almost a constant percentage of reading for a constant current or constant power test. The power capability uncertainty curves are more complex, because they are a combination of effects of statistical and calibration errors whose relative magnitudes are data-dependent. In effect, the shapes of these curves depend on the shapes of the power capability curves, which in turn vary, depending on the specific device under test. Thus, there is no general form of these relationships; they must be determined for each data set.

Let the three uncertainty relationships in Figures 10 and 11 be represented by functions $U_E(Q)$, $U_{D,P}(Q)$, and $U_{R,P}(Q)$. For example, the uncertainty of regen power P_R (i.e., the regen power uncertainty at DOD value Q_R in Figure 11) is denoted by $U_{R,P}(Q_R)$.

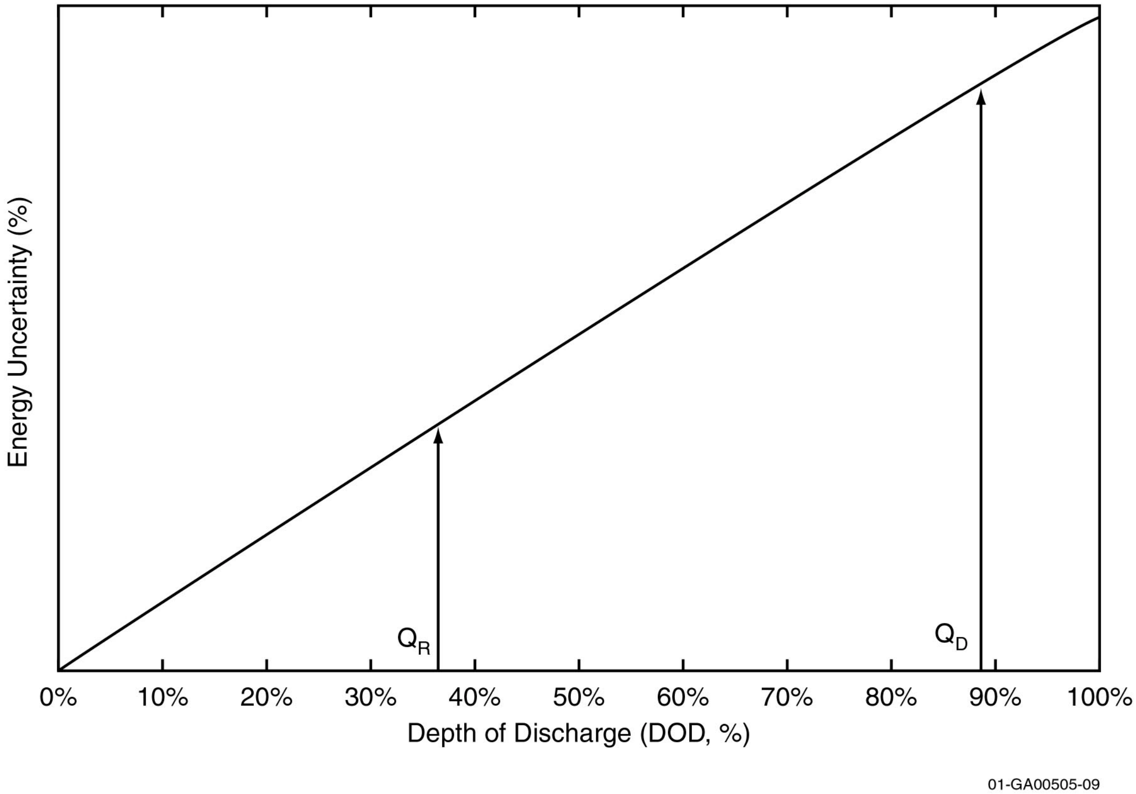

Figure 10. Example energy uncertainty as a function of DOD.

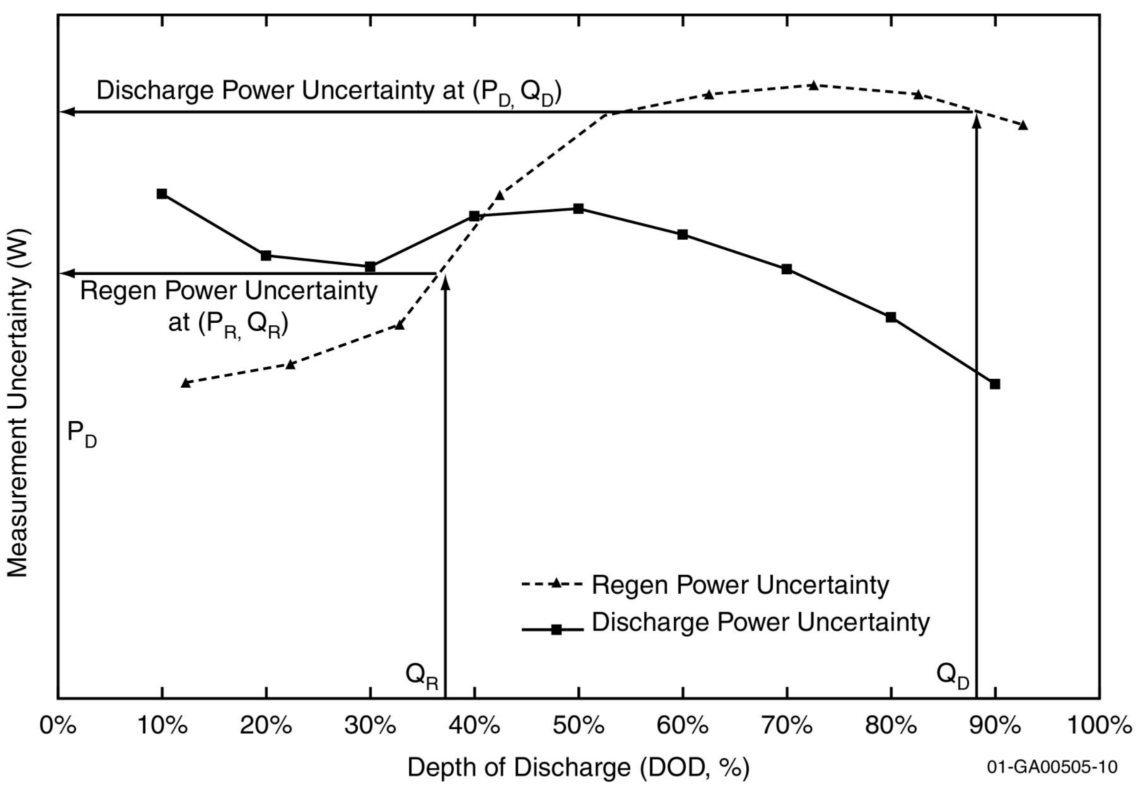

Figure 11. Example power capability uncertainties as a function of DOD.

Now, $\Delta E_{R,P}$, the error in E_R due to uncertainty in P_R, can be determined based on the following logic: the uncertainty in P_R [i.e., $U_{R,P}$ (Q_R)] implies some corresponding uncertainty in Q_R.[r] This uncertainty in Q_R due to P_R will in turn affect the calculated uncertainty of energy value E_R according to the relationship in Figure 10. An error in Q_R of magnitude ΔQ_R will result in a change in the uncertainty of E_R of magnitude $U_E(Q_R+\Delta Q_R) - U_E(Q_R)$. This change in the uncertainty of E_R is the error term $\Delta E_{R,P}$. We assume (a) that all the functions defined above are continuous and "well behaved," and (b) that the uncertainties involved are small fractions of the parameter values.[s] This allows the functions to be treated as linear in a local region about the point of interest. For example, Figure 12 shows the result near Q_R of fitting a curve to the calculated regen power uncertainty values in Figure 11. Even though the curve fit uses a sixth-order polynomial approximation, the result is still approximately linear in the region around Q_R.

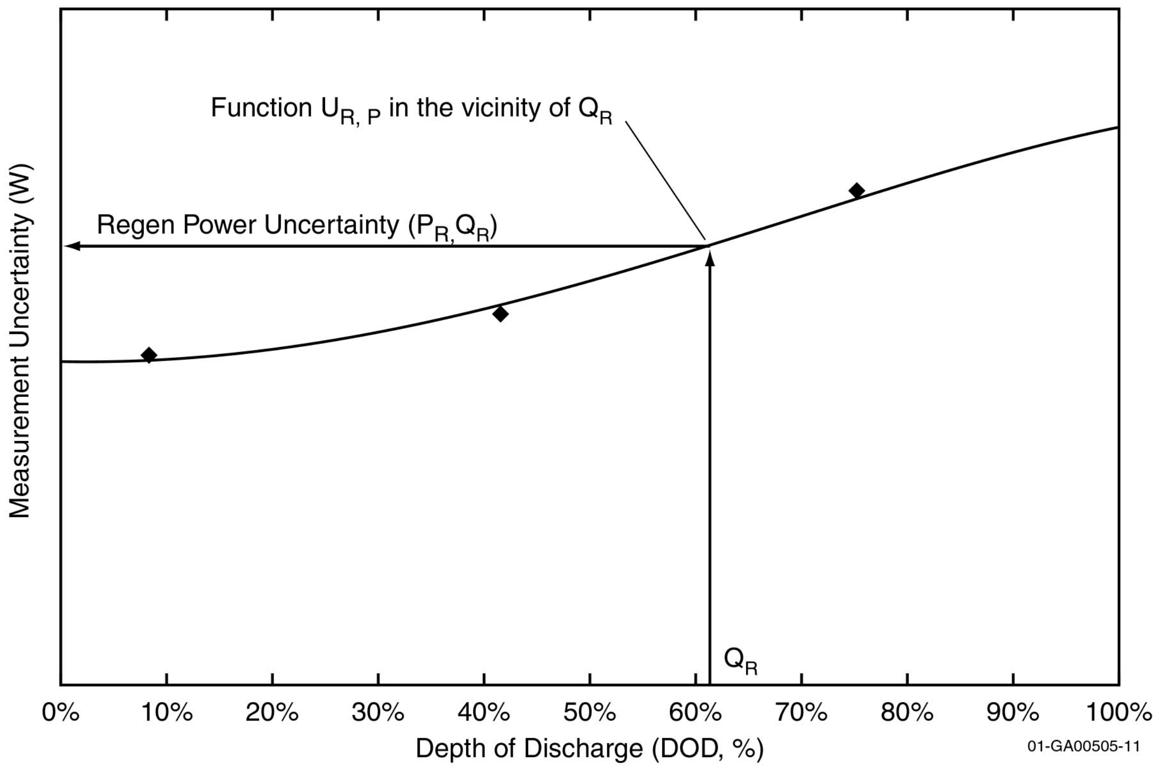

Figure 12. Example of approximate linear behavior of uncertainty function.

r. The implied uncertainty in Q_R referred to here is *not* due to the measurement (actually calculation) of DOD, which is being neglected as previously noted. Instead, the existence of regen power uncertainty is interpreted to mean that some value of Q different from Q_R could be the value that actually corresponds to P_R, and the range of such values of Q is treated as an uncertainty in Q_R.

s. Assumption a is almost guaranteed to be true because (1) all the functions related to power capability are determined by fitting curves through no more than nine data points, typically using low-order polynomial approximations, and (2) the energy versus DOD curve and the associated energy uncertainty curve are approximately linear. Assumption b is not guaranteed, but the acceptability of very large uncertainties is not likely to depend on their exact values, in any event.

Using these assumptions yields the following results:

$$\Delta E_{R,P} = \Delta Q_R \cdot \frac{dU_E}{dQ}(Q_R)$$

where $\frac{dU_E}{dQ}(Q_R)$ is the slope of the function U_E at Q_R[t]

$$\Delta Q_R = U_{R,P}(Q_R) \cdot \frac{dh}{dP}(P_R)$$

where $\frac{dh}{dP}(P_R)$ is the slope of the function h at P_R.

The error in energy value E_R (in units of energy) as a result of the measurement uncertainty associated with P_R is thus defined by Equation (101):

$$\Delta E_{R,P} = U_{R,P}(Q_R) \cdot \frac{dh}{dP}(P_R) \cdot \frac{dU_E}{dQ}(Q_R) \ . \quad (101)$$

In exactly analogous fashion, the error in energy value E_D (in units of energy) as a result of the measurement uncertainty associated with P_D is defined by Equation (102):

$$\Delta E_{D,P} = U_{D,P}(Q_D) \cdot \frac{dg}{dP}(P_D) \cdot \frac{dU_E}{dQ}(Q_D). \quad (102)$$

Substituting Equations (99), (101), and (102) into Equation (100) gives the result of Equation (103) in units of energy.

$$\Delta AE = \left\{ \begin{array}{c} \left(\left[\int I dt \right] V_{FS} \frac{\%err_{VCAL}}{100} \right)^2 + \left(\left[\int V dt \right] I_{FS} \frac{\%err_{ICAL}}{100} \right)^2 + \\ \left(U_{D,P}(Q_D) \cdot \frac{dg}{dP}(P_D) \cdot \frac{dU_E}{dQ}(Q_D) \right)^2 + \\ \left(U_{R,P}(Q_R) \cdot \frac{dh}{dP}(P_R) \cdot \frac{dU_E}{dQ}(Q_R) \right)^2 \end{array} \right\}^{\frac{1}{2}} . \quad (103)$$

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t. Note that all these expressions are written as though the slopes of these curves are positive. This can be done without loss of generality because the resulting expressions will be combined in RSS fashion using Equation (100), so the signs of the individual terms are irrelevant.

This result is converted from units of energy to percentage of reading by dividing by the calculated available energy and multiplying by 100 to give the final result of Equation (104):

$$\%AE = \frac{100}{AE}\left\{\begin{array}{c}\left(\left[\int Idt\right]V_{FS}\frac{\%err_{VCAL}}{100}\right)^2 + \left(\left[\int Vdt\right]I_{FS}\frac{\%err_{ICAL}}{100}\right)^2 + \\ \left(U_{D,P}(Q_D)\cdot\frac{dg}{dP}(P_D)\cdot\frac{dU_E}{dQ}(Q_D)\right)^2 + \\ \left(U_{R,P}(Q_R)\cdot\frac{dh}{dP}(P_R)\cdot\frac{dU_E}{dQ}(Q_R)\right)^2\end{array}\right\}^{\frac{1}{2}} . \qquad (104)$$

where

P_D	=	PNGV Discharge Pulse Power Capability goal
P_R	=	PNGV Regen Pulse Power Capability goal
Q_D	=	battery state (DOD) at which the calculated Discharge Power Capability is exactly equal to the PNGV goal. (This is the maximum depth of discharge at which both PNGV power goals can be met.)
Q_R	=	battery state (DOD) at which the calculated Regen Power Capability is exactly equal to the PNGV goal. (This is the minimum depth of discharge at which both PNVG power goals can be met.)
$\int Idt$	=	integral of current between the battery (DOD) states Q_R and Q_D
$\int Vdt$	=	integral of voltage between the battery (DOD) states Q_R and Q_D
$U_{D,P}(Q_D)$	=	Discharge Power Uncertainty vs DOD function evaluated at Q_D. (Function is determined empirically from test data at various DOD values.)
$U_{R,P}(Q_R)$	=	Regen Power Uncertainty vs DOD function evaluated at Q_R. (Function is determined empirically from test data at various DOD values.)
g	=	function relating battery state (DOD) to Discharge Power Capability (determined empirically from test data taken at various DOD values).
$\frac{dg}{dP}(P_D)$	=	slope (derivative) of function g at P_D
h	=	function relating battery state (DOD) to Regen Power Capability (determined empirically from test data taken at various DOD values).
$\frac{dh}{dP}(P_R)$	=	slope (derivative) of function h at P_R

U_E = function relating Energy Uncertainty to DOD (determined empirically from test data taken at various DOD values).

$\frac{dU_E}{dQ}(Q_D)$ = slope (derivative) of function U_E at Q_D

$\frac{dU_E}{dQ}(Q_R)$ = slope (derivative) of function U_E at Q_R

V_{FS} = full-scale voltage

I_{FS} = full-scale current

$\%err_{ICAL}$ = current calibration error (as $\%FS$)

$\%err_{VCAL}$ = voltage calibration error (as $\%FS$).

Note that the last two terms in Equation (104) cannot be calculated directly from the original test data. All the functions involved are based on curves fitted to a particular set of test data. The nature of this functionality (i.e., the shape or form of these curves) varies with the device being tested. Thus, arriving at a numerical value for the uncertainty of available energy for any given data set requires a great deal more calculation than is necessary for other derived parameters. However, the assumption of "local near-linearity" used for deriving Equations (101) and (102) can be used to advantage in simplifying these calculations in many cases. Both the derivatives and the uncertainty functions in these equations can be estimated by linear approximations using the nearest calculated points, rather than actually performing the curve fitting. This approach is specifically recommended for automated calculations; experience has shown that the power capability results in particular are so variable in form that automated curve fitting is risky. Further, there are so many sources of variability in this process (due to the various assumptions and the multiple levels of indirect calculations) that the error added by linear approximations is not likely to be dominant. The result of this calculation probably should not be given the same degree of confidence (in the general, not the statistical, sense) allowed to the other derived parameters.

4. SUMMARY

This first volume of the overall report derives uncertainty relationships for those measured and derived parameters important to INEEL high-power battery testing. These relationships can be used to calculate measurement uncertainties for a battery test station or other measurement devices using these same parameters and a comparable measurement approach. The relationships for directly measured parameters (temperature, current, and voltage) are generally expressed as a percentage of the full-scale range of the measurement channel. Relationships for derived parameters are commonly a function of the actual values of one or more direct measurements, and the concept of measurement range is not meaningful for many of them, especially where time integrals are involved. Thus, the derived parameter uncertainties are generally given as a percentage of reading. To apply these relationships, some combination of manufacturer's specifications, design information, and uncertainty test results will generally be required. Note that the results are all expressed in terms of a standard deviation, and an appropriate multiplier must be used to give the desired confidence level for the results (e.g., 2x for 95% confidence).

One or more subsequent volumes of this report will be prepared to apply these relationships to specific types of INEEL battery laboratory test stations and to document the testing that will be required to supplement or confirm manufacturer's uncertainty specifications. Note that this volume, Volume 1, develops the theoretical background for aliasing and defines aliasing errors. However, the actual potential for aliasing in INEEL test stations depends on equipment design and application; thus, the issue of whether aliasing error must be included in uncertainty calculations is treated later.