

## Extended Simplified Model Explanation

The Extended Simplified Model estimates the battery cell capacity and number of cells needed to satisfy PNGV battery requirements. An Excel spreadsheet is used to implement the ESM. This report will explain variables and describe the use and origin of the equations used to accomplish the ESM goals. Data set and constant definitions will indicate if they need to be carried in from outside sources.

The ESM begins by creating arrays for power using Equations 1 and 2; the equations are described in greater detail in *PNGV Battery Test Manual Revision 3*, but the variable names are somewhat different.  $R$  and  $OCV$  are measured at certain states of charge, and for each  $SOC$  they are used to calculate  $P_{REF, D}$  and  $P_{REF, R}$ .

$$P_{REF, D} = V_{MIN, C} \cdot \frac{OCV\{SOC\} - V_{min}}{R_{REF, D}\{SOC\}} \quad (1)$$

$$P_{REF, R} = V_{MAX, C} \cdot \frac{V_{MAX, C} - OCV\{SOC\}}{R_{REF, R}\{SOC\}} \quad (2)$$

$V_{min}$  = minimum cell voltage for power capability (maximum of  $V_{MIN, C, ABS}$  or  $V_{MIN, C}$ )

$V_{MAX} =$  maximum desired battery (system) voltage

$V_{MIN} =$  minimum desired battery (system) voltage

$V_{MAX, C} =$  maximum allowable cell voltage (given by manufacturer)

$V_{MIN, C, ABS} =$  absolute minimum allowable cell voltage (given by manufacturer)

$V_{MIN, C} =$   $V_{MAX, C} \times (V_{MIN} / V_{MAX})$  (minimum allowable cell voltage relative to PNGV requirements)

$OCV\{SOC\} =$  open circuit voltage data set (carried in from HPPC test)

$R_{REF, D}\{SOC\} =$  discharge resistance data set (carried in from HPPC test)

$R_{REF, R}\{SOC\} =$  recharge resistance data set (carried in from HPPC test)

$P_{REF, D}\{SOC\} =$  discharge pulse power data set (calculated in Equation 1)

$P_{REF, R}\{SOC\} =$  recharge pulse power data set (calculated in Equation 2)

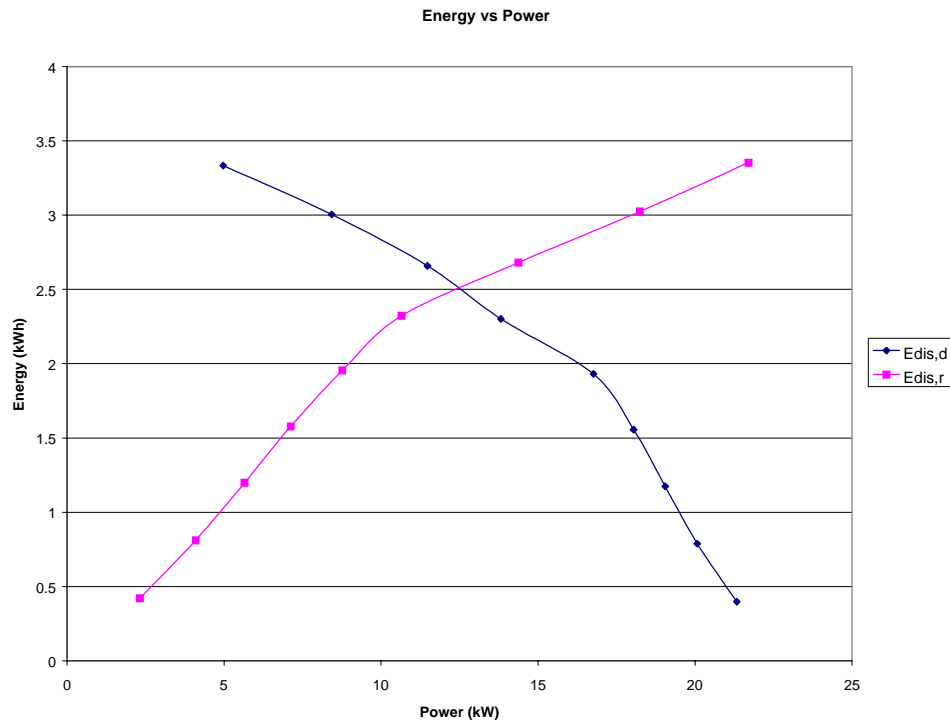
From a C/1 discharge test, a data set of energy vs. SOC can be carried in.

$E_{DIS, D}\{SOC\}, =$  energy discharged from full charge to any SOC (this information is used

$E_{DIS, R}\{SOC\}$  twice, so it is named twice)

Since both power and energy can be separately plotted against SOC, a plot of power vs. SOC can be re-scaled to become a plot of power vs. energy. Then for convenience the axes can be switched to create a plot of energy vs. power. This is done to the data sets of  $P_{REF, D}$  and  $P_{REF, R}$ ;  $E_{DIS, D}$  and  $E_{DIS, R}$  label the curves in Figure 1.

**Figure 1: Energy vs. Power**



For the purpose of describing battery behavior, assume that the data used to plot the two curves in Figure 1 can be represented by the second order polynomials in Equations 3 and 4.<sup>1</sup> A regression technique is used to fit the polynomials to the data sets and determine the coefficients to these equations as seen in Cases 1 and 2 below.<sup>2</sup> For each case, energy is assumed to be the dependent y-value while power and [power squared] are the independent  $x_1$ -values.

<sup>1</sup> This is not always a good assumption, because a second-order polynomial may not be adequate to represent the data set with sufficiently high quality. However, it is generally good enough for the intended purpose of estimating the required number of cells and cell capacity needed to meet the PNGV goals under some set of conditions.

<sup>2</sup> For the “ESM\_Example\_Spreadsheet.XLS” provided with the PNGV testing manual, the EXCEL function named LINEST is used to perform multivariable linear regressions. The user first highlights a single column of data representing known dependent y values that correspond to the relationship  $y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 \dots$

$$E_{DIS,D} = A_D + B_D \cdot P_{REF,D} + C_D \cdot P_{REF,D}^2 \quad (3)$$

$$E_{DIS,R} = A_R + B_R \cdot P_{REF,R} + C_R \cdot P_{REF,R}^2 \quad (4)$$

<p>Case 1: <math>y = E_{DIS,D}</math>  <math>x_1 = P_{REF,D}</math>  <math>x_2 = (P_{REF,D})^2</math>  <math>a_0 = A_D</math>  <math>a_1 = B_D</math>  <math>a_2 = C_D</math></p>	<p>Case 2: <math>y = E_{DIS,R}</math>  <math>x_1 = P_{REF,R}</math>  <math>x_2 = (P_{REF,R})^2</math>  <math>a_0 = A_R</math>  <math>a_1 = B_R</math>  <math>a_2 = C_R</math></p>
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From these polynomial equations for  $E_{DIS,D}$  and  $E_{DIS,R}$ , a method for estimating the required cell capacity and number of cells is developed. The method is designed to satisfy certain PNGV-defined constraints, including the ratios  $Pr/Pd$  and  $Pd/E_{AVAIL}$ , where

$Pr$  = maximum regenerative pulse power required (given in PNGV battery test manual)  
 $Pd$  = maximum discharge pulse power required (given in PNGV battery test manual)  
 $E_{AVAIL}$  = available energy

In Figure 1, the useful discharge region of a battery is the area below the  $E_{DIS,D}$  curve and above the  $E_{DIS,R}$  curve, which represent energy (and DOD) limits with respect to power. Graphically, this means that the maximum “usable energy”<sup>3</sup>, for some power level, would be the vertical distance between  $E_{DIS,D}$  and  $E_{DIS,R}$ . (Note: the vertical distance between the two curves to the right of their intersection in Figure 1 would result in negative energy, so it is disregarded.)

Available Energy is defined as the energy discharged between the PNGV-defined power goal constants  $Pd$  and  $Pr$ . ( $Pd$  lies on the  $E_{DIS,D}$  curve and  $Pr$  lies on the  $E_{DIS,R}$  curve.) However,  $Pd$  and  $Pr$  are not equal; symbolically, this means that to find Available Energy one must subtract a function of regen power [ $f_2(P_{REF,R}) = E_{DIS,R}(P_{REF,R})$ ] from a function of discharge power [ $f_1(P_{REF,D}) = E_{DIS,D}(P_{REF,D})$ ].

As mentioned earlier, the methodology used will only concern itself with the ratio  $Pr/Pd$ . This way, a general equation for usable energy for any power level can be created based on Equations 3 and 4. There still exists the problem of regen and discharge power not being equal. Fortunately, it is known that for any energy calculation, the ratio of these powers will be  $Pr/Pd$ .

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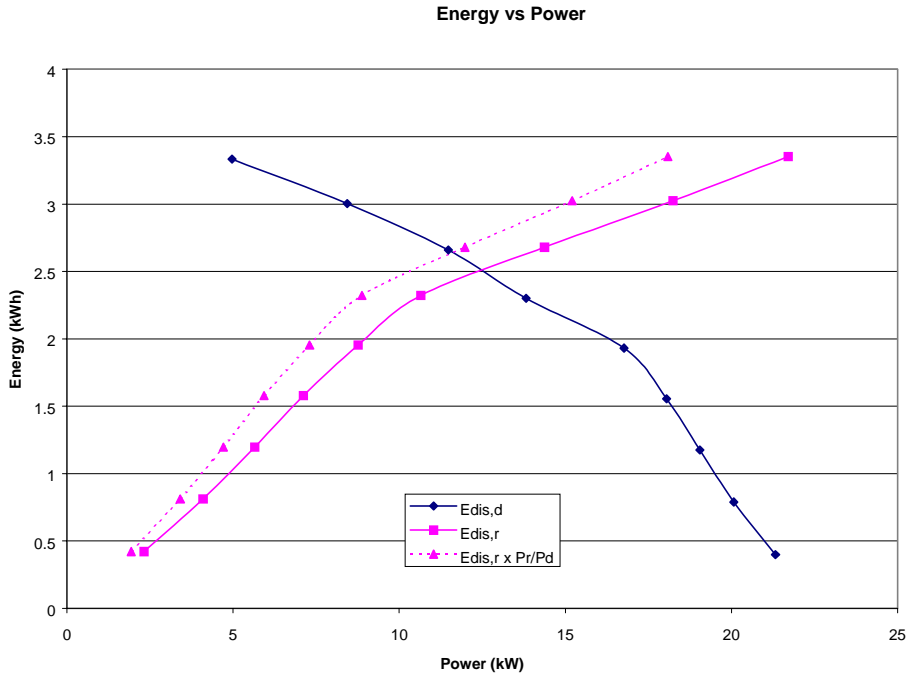
Any number of other columns can be designated as independent  $x_i$  values. The LINEST function will then compute the coefficients  $a_0, a_1, a_2, a_3$ , etc. This technique is used here by the expedient of treating  $P$  and  $P^2$  as independent variables.

<sup>3</sup> The term “usable energy” is used in this document to describe generally that region of the energy/power space shown in Figure 1 which is below the discharge curve and above the regen curve. This is the operating area where some constraint on discharge pulse power and regen pulse power can be satisfied simultaneously. The term “available energy” has sometimes been used for this region, but “Available Energy” for PNGV is the “usable energy” corresponding exactly to the PNGV system power goals, i.e., there is only one value of available energy for a given data set.

The calculation can be simplified by transforming the regen function so that it is expressed in terms of discharge power, so that each value for  $P_{REF,R}$  in effect lies directly below its corresponding value for  $P_{REF,D}$ . In this way only discharge power is used when calculating energy (i.e., by subtracting  $f_2^*(P_{REF,D})$  from  $f_1(P_{REF,D})$ ). This situation can be achieved by substituting  $[P_{REF,D} \cdot (Pr/Pd)]$  for  $P_{REF,R}$  in Equation 4 to create Equation 5. Making this substitution will cause the regen curve to shift to the left as seen in Figure 2. Symbolically this is a change of independent variable where  $f_2^*(P_{REF,D}) = f_2(k P_{REF,D}) = f_2(P_{REF,R})$ ,  $k = Pr/Pd$ , and  $(k P_{REF,D}) = P_{REF,R}$ . The independent variable in Equation 4 was  $P_{REF,R}$ , but the independent variable in Equation 5 is now  $P_{REF,D}$ .

$$E_{DIS,R} = A_R + B_R \cdot P_{REF,D} \cdot (Pr/Pd) + C_R \cdot P_{REF,D}^2 \cdot (Pr/Pd)^2 \quad (5)$$

**Figure 2: Energy vs. Power Rescaled**



Notice that the vertical position of each data point has not changed, and each discharge power is now directly above its corresponding regen power. Taking the difference between the equations for  $E_{DIS,D}$  (3) and  $E_{DIS,R}$  (5) creates an equation for usable energy for any given discharge power (6).

$$E = A_A + B_A \cdot P_{REF,D} + C_A \cdot P_{REF,D}^2 \quad (6)$$

$E =$  usable energy for a given power level

$$A_A = A_D - A_R$$

$$B_A = B_D - B_R (Pr/Pd)$$

$$C_A = C_D - C_R (Pr/Pd)^2$$

To find battery capacity and number of cells, a constant named *Battery Sizing Parameter* ( $NC/E$ ) is defined. The derivation of  $NC/E$  begins with Equation 6. Remember that Equation 6 is a general equation that finds the maximum usable energy for any given discharge power. For convenience,  $Pd$  is abbreviated as  $P$ .

$NC/E =$  normalized battery sizing parameter (Ah/Wh)

$P/E =$  nominal power/energy ratio (calculated as  $Pd/E_{AVAIL}$ )

To derive the Battery Sizing Parameter, Equation 6 will be reformulated in terms of  $P/E$ , by multiplying certain parts of the equation by  $E/E$  and rearranging.

$$E = A_A + B_A \cdot P \cdot \left(\frac{E}{E}\right) + C_A \cdot P^2 \cdot \left(\frac{E}{E}\right)^2$$

$$E = A_A + B_A \cdot \left(\frac{P}{E}\right) \cdot E + C_A \cdot \left(\frac{P}{E}\right)^2 \cdot E^2 \quad (7)$$

Get the equation into quadratic form.

$$-C_A \cdot \left(\frac{P}{E}\right)^2 \cdot E^2 + \left(1 - B_A \left(\frac{P}{E}\right)\right) \cdot E - A_A = 0 \quad (9)$$

Assuming Equation 9 is in quadratic form, the following can be declared.

$$a \cdot x^2 + b \cdot x + c = 0 \quad \text{where :}$$

$$a = -C_A \cdot \left(\frac{P}{E}\right)^2$$

$$b = 1 - B_A \cdot \left(\frac{P}{E}\right)$$

$$c = -A_A$$

$$x = E$$

Using the quadratic formula, Equation 9 can be solved for  $E$  as in Equation 10.<sup>4</sup> (The other solution is discarded as not corresponding to the conditions of interest.)

$$E = \frac{-(1 - B_A \cdot (P/E)) + \sqrt{1 - 2 \cdot B_A \cdot (P/E) + (B_A^2 - 4 \cdot A_A \cdot C_A) \cdot (P/E)^2}}{-2 \cdot C_A \cdot (P/E)^2} \quad (10)$$

<sup>4</sup> Note that this equation is self-referential, i.e., it calculates  $E$  using  $E$ . In practice the ratio  $P/E$  is fixed in order to be able to perform this calculation.

The normalized battery sizing factor is now derived by inverting this expression and multiplying it by the known cell capacity.

$AH_{REF}$  = cell capacity measured at a specified discharge rate

The resulting factor,  $NC/E$ , is thus found by the expression

$$\frac{NC}{E} = \frac{-2 \cdot C_A \cdot (P/E)^2 \cdot AH_{REF}}{\sqrt{1 - 2 \cdot B_A \cdot (P/E) + (B_A^2 - 4 \cdot A_A \cdot C_A) \cdot (P/E)^2} - (1 - B_A \cdot (P/E))} \quad (11)$$

$NC/E$  will now be used to determine cell capacity and the number of cells required to satisfy PNGV battery requirements.

Use the more restrictive of  $V_{MIN, C}$  and  $V_{MIN, C, ABS}$  (i.e., whichever is greater) and call it  $V_{min}$ . Now the cell capacity and required number of cells can be estimated; capacity is represented by

$AH_{RATED}$  = cell rated capacity

Divide the minimum voltage for a battery by the minimum voltage for a single cell to estimate the required number of cells  $N_{SERIES}$ , and round up to the nearest integer.

$$N_{SERIES} = \text{INTEGER} \left\{ \frac{V_{MIN}}{V_{min}} \right\} + 1 \quad (12)$$

Based on  $N_{SERIES}$ , make an estimate for  $AH_{RATED}$  using Equation 13. (This is multiplied by 1000, because  $E$  is in units of kWh.)

$$AH_{RATED} = 1000 \cdot (NC/E) \cdot (E_{AVAIL} / N_{SERIES}) \quad (13)$$

Be sure that  $AH_{RATED}$  satisfies the following constraint.

$$AH_{MIN} \leq AH_{RATED} \leq AH_{MAX} \quad \text{where :}$$

$AH_{MIN}$  = minimum rated cell capacity (given by manufacturer)

$AH_{MAX}$  = maximum rated cell capacity (given by manufacturer)

If  $AH_{RATED}$  goes out of bounds, set it equal to whatever boundary was violated.

Now  $N$  can be re-estimated using Equation 14. Notice that it is Equation 13 solved for  $N_{SERIES}$ .

$$N_{SERIES} = \text{INTEGER} \left\{ 1000 \cdot (NC/E) \cdot (E_{AVAIL} / AH_{RATED}) \right\} \quad (14)$$

If  $AH_{RATED}$  was equal to  $AH_{MAX}$ , then the original  $N_{SERIES}$  was too small, so add 1 to the result above and recalculate Equation 13 to find  $AH_{RATED}$ .

Note that  $N_{SERIES}$  is now an integer that may have been increased by some number, so the actual system minimum voltage is now  $N_{SERIES} \cdot V_{min}$ , but this may not be equal to the minimum desired system voltage  $V_{MIN}$ .